SLACKISH BUSINESS-CYCLE MODEL: STATIC VERSION

Pascal Michaillat

December 2023

Course material available at https://pascalmichaillat.org/c5/

OUTLINE

- Present a slackish model of business cycles
  - Static version
  - Based on Michaillat, Saez (2015)
- Solve the model
- Study impact of aggregate demand and supply shocks
- Compare predictions of different price norms

KEY MODELING ASSUMPTIONS

- Static model $\rightsquigarrow$ comparative statics
- All workers are self-employed $\rightsquigarrow$ only one market
- All trade mediated by matching function $\rightsquigarrow$ slack
- Wealth in the utility function $\rightsquigarrow$ nondegenerate aggregate demand
- Unequal income and wealth $\rightsquigarrow$ inequality and theoretical clarity (different aggregate and individual variables)
- Prices determined by rigid price norm $\rightsquigarrow$ realism & simplicity

MATCHING MARKET AND MATCHING FUNCTION

MATCHING MARKET

- S sellers and B buyers exchanging a type of good
- Trading process is complex:
  - All goods are somewhat different
  - All buyers have different preferences
  - Marketplace is not centralized
- Number of trades determines by matching function (less than S and B)
- Price are determined by price norm in bilateral-monopoly situations
- In practice, almost all markets are matching markets:
  - Labor market
  - Market for services and many goods
- Auction markets are the exception (stock market, commodity markets)

MATCHING FUNCTION (PETRONGOLO, PISSARIDES 2001)

- The matching function m is an aggregate function that summarizes a complex process that occurs on most markets
  - Just like a production function is an aggregate function that summarizes a complex production process
- In markets with one good per person: $M = m(S, B)$
  - M: number of trades (labor market example: hires)
  - S: number of sellers (labor market example: jobseekers)
  - B: number of buyers (labor market example: vacancies)
  - With $m(S, B) \leq \min\{S, B\}$

PROPERTIES OF MATCHING FUNCTION (PETRONGOLO, PISSARIDES 2001)

- Need some buyers and sellers for trades

$$m(0, B) = m(S, 0) = 0$$

- More sellers or buyers lead to more trades

$$\frac{\partial m}{\partial S} > 0 \quad \text{and} \quad \frac{\partial m}{\partial B} > 0$$

- Constant returns to scale (for tractability, and realistic)

$$m(\lambda \cdot S, \lambda \cdot B) = \lambda \cdot m(S, B)$$

- Concavity in both arguments: $\partial^2 m / \partial S^2 < 0$ and $\partial^2 m / \partial B^2 < 0$

MARKET TIGHTNESS

- Market tightness is a key aggregate variable on any matching market
- Market tightness is as important as the price (in some ways more important)
- Market tightness is the ratio of buyers to sellers: $\theta = B/S$
  - Labor market example: θ = vacancies / jobseekers
- Tightness is so important because it determines all trading probabilities:
  - Probability to sell a good
  - Probability to buy a good
  - (Which are assumed to be 100% in competitive markets)

SELLING PROBABILITY

$$f(\theta) = \frac{M}{S} = \frac{m(S, B)}{S} = m\left(\frac{S}{S}, \frac{B}{S}\right) = m(1, \theta)$$

- Selling probability only depends on tightness
- No chance of selling without buyers: $f(0) = 0$
- Selling probability is increasing in tightness: $f'(\theta) > 0$
  - Tight markets are favorable to sellers
  - When there are a lot of buyers relative to sellers, it is easy to sell
- Selling probability is concave in tightness: $f''(\theta) < 0$

BUYING PROBABILITY

$$q(\theta) = \frac{M}{B} = \frac{m(S, B)}{B} = m\left(\frac{S}{B}, \frac{B}{B}\right) = m\left(\frac{1}{\theta}, 1\right)$$

- Buying probability only depends on tightness
- No chance of buying without sellers: $q(\infty) = 0$
- Buying probability is decreasing in tightness: $q'(\theta) < 0$
  - Tight markets are unfavorable to buyers
  - When there are a lot of buyers relative to buyers, it is difficult to buy

RELATIONSHIP BETWEEN SELLING AND BUYING PROBABILITIES

$$f(\theta) = m(1, \theta) = \theta \cdot m\left(\frac{1}{\theta}, 1\right) = \theta \cdot q(\theta)$$

- Result obtained by constant returns to scale
- Means that $f(\theta) = \theta \cdot q(\theta)$
- Also means that $\theta = f(\theta)/q(\theta)$

COBB-DOUGLAS MATCHING FUNCTION

$$m(S,B) = \omega \cdot S^{\eta} \cdot B^{1-\eta}$$

- ω: matching efficacy
- η: matching elasticity
- $f(\theta) = m(1,\theta) = \omega \cdot \theta^{1-\eta}$ with $\theta = B/S$
- $q(\theta) = m(1/\theta, 1) = \omega \cdot \theta^{-\eta}$ with $\theta = B/S$
- Realistic matching function on the labor market (Petrongolo, Pissarides 2001)
  - $\eta \in [0.5, 0.7]$, commonly calibrated to $\eta = 0.5$
- Works well in continuous time, where $m(S,B)$ gives a matching rate
- But inconvenient in discrete-time matching models
  - Requires to add the constraint $m(S,B) \leq \min\{S,B\}$

CONSTANT-ELASTICITY-OF-SUBSTITUTION (CES) MATCHING FUNCTION

$$m(S, B) = \left[S^{-\gamma} + B^{-\gamma}\right]^{-1/\gamma}$$

- $\gamma > 0$: governs the elasticity of substitution
- $f(\theta) = m(1, \theta) = \left[1 + \theta^{-\gamma}\right]^{-1/\gamma}$ with $\theta = B/S$
- $q(\theta) = m(1/\theta, 1) = \left[1 + \theta^{\gamma}\right]^{-1/\gamma}$ with $\theta = B/S$
- Convenient matching function in discrete-time models
  - Always satisfies $m(S, B) < \min\{S, B\}$
- But not realistic empirically as it implies highly nonconstant matching elasticity

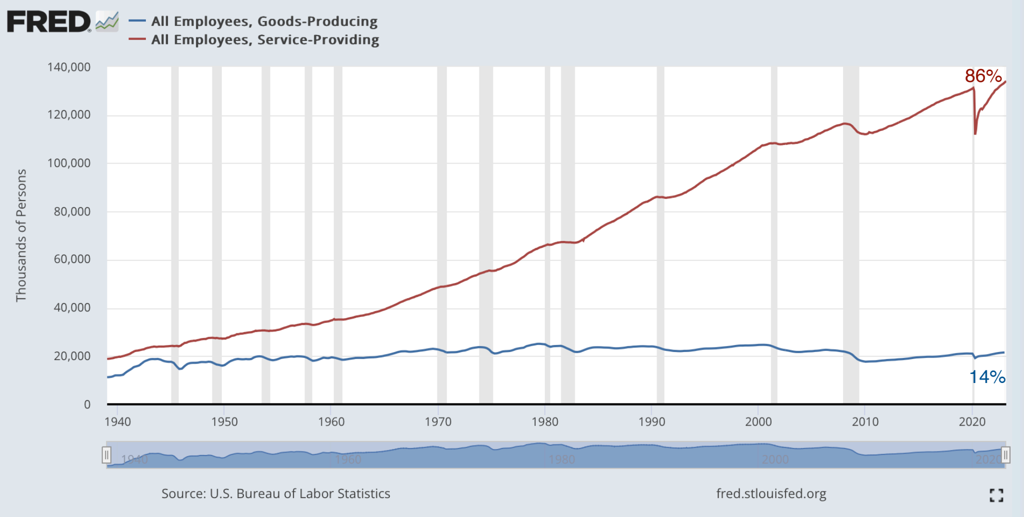
$$\eta(\theta) = \frac{\partial \ln(m)}{\partial \ln(S)} = \frac{1}{1 + \theta^{-\gamma}}$$

DESCRIPTION OF THE MODEL

STRUCTURE

- Static, one-period model
- Households are self-employed and produce services
  - No firms, so no distinct labor and product markets
  - Only one market for services
- Households purchases and consume services produced by other households
- All services are traded on a matching market
  - Trades mediated by a CES matching function
- Households are endowed with wealth
  - Used as numeraire

MOST EMPLOYMENT PRODUCES SERVICES



WEALTH INEQUALITY

- Each household $i \in [0, 1]$ starts with endowment of wealth μ_i
- Aggregate wealth: $\mu = \sum \mu_i$
- Wealth may be money, land, gold, art
  - Any good that is nonproduced and valuable
- Wealth is used as numeraire
  - All prices are expressed in units of wealth
- Introducing wealth is key to obtain a nondegenerate aggregate demand
  - Households must choose between consumption and something else

INEQUALITY IN PRODUCTIVE CAPACITY

- Each household $i \in [0, 1]$ offers k_i services to the market:
  - Reflects inequality in human capital, productive capital, taste for work and leisure
  - Produces income inequality
- Aggregate capacity: $k = \sum k_i$
- Capacity is fixed, reflecting the capacity is acyclical in the data:
  - Acyclical technology
  - Acyclical capital stock
  - Acyclical labor-force participation

HOUSEHOLD UTILITY FUNCTION

$$\mathcal{U}\left(c_i, \frac{m_i}{p}\right) = \frac{\chi}{1+\chi} c_i^{\frac{\epsilon-1}{\epsilon}} + \frac{1}{1+\chi}\left(\frac{m_i}{p}\right)^{\frac{\epsilon-1}{\epsilon}}$$

- Household i consumes c_i services
- Household i holds m_i / p units of real wealth
  - m_i: nominal wealth holdings
  - p: aggregate price level
  - $m = \sum m_i$: aggregate nominal wealth holdings
- $\chi > 0$: taste for services relative to wealth
- $\epsilon > 1$: elasticity of substitution between consumption and wealth

JUSTIFICATION FOR WEALTH IN UTILITY FUNCTION

- People enjoy accumulating wealth in itself, not for future consumption:

  > *The duty of saving became nine-tenths of virtue and the growth of the cake the object of true religion… Saving was for old age or for your children; but this was only in theory—the virtue of the cake was that it was never to be consumed, neither by you nor by your children after you. (Keynes 1919)*

- Many reasons why people enjoy wealth, including social status and power:

  > *A man may include in the benefits of his wealth… the social standing he thinks it gives him, or political power and influence, or the mere miserly sense of possession, or the satisfaction in the mere process of further accumulation. (Fisher 1930)*

- Neuroscientific evidence: wealth itself provides utility (Camerer, Loewenstein, Prelec 2005)

MATCHING PROCESS

- Household i visits v_i shops to buy services from other households
- Household i offers k_i services for sale
- Matching function determines number of services sold and purchased:

$$y = m\left(\sum k_i, \sum v_i\right)$$

  - y: output
  - $k = \sum k_i$: aggregate capacity
  - $v = \sum v_i$: aggregate number of visits

MARKET TIGHTNESS AND TRADING PROBABILITIES

- Market tightness:

$$x = \frac{\sum v_i}{\sum k_i} = \frac{v}{k}$$

- Probability to sell one service: $f(x) = m(1, x)$
- Services sold by household i: $f(x)k_i$
- Probability to buy a service in a visit: $q(x) = m(1/x, 1)$
- Services purchased by household i: $q(x)v_i$

SHOPPING COST

- Each visit requires $\kappa \in (0, 1)$ services
- Service purchased > services consumed
  - Additional purchased services used for shopping
- Why do we need to introduce a cost of shopping?
  - Theoretical symmetry: it's costly for sellers to spend their day in the shop waiting for customers, so on both sides of the market finding a trading partner is costly
  - Provides an interior solution to welfare problem
  - Sellers and buyers are both generally happy to trade, indicating that they derive a surplus from trading and thus that there are costs
  - Realism: it's costly to shop for sellers (time, effort, middlemen, brokers)

SHOPPING WEDGE: GAP BETWEEN PURCHASES AND CONSUMPTION

- Household conducts v visits to consume c services
- This requires to purchase $c + \kappa v$ services
- One visit yields $q(x)$ services, so 1 purchase requires $1/q(x)$ visits, and so the household's visits satisfy:

$$
\begin{aligned}
v &= \frac{c}{q(x)} + \kappa \frac{v}{q(x)} \\
v\left(1 - \frac{\kappa}{q(x)}\right) &= \frac{c}{q(x)} \\
v\,(q(x) - \kappa) &= c \\
\kappa v &= c \frac{\kappa}{q(x) - \kappa}
\end{aligned}
$$

PROPERTIES OF THE SHOPPING WEDGE

- $\kappa v = c\kappa/[q(x) - \kappa]$ is the number of services required to shop for c services consumed. The shopping wedge is the number of shopping services required for or one service consumed:

$$\tau(x) = \frac{\kappa}{q(x) - \kappa}$$

- Consuming 1 service requires to purchase $1 + \tau(x)$ services
  - Akin to a firm employing human-resource workers to hire producers
- $\tau(0) = \kappa/(1 - \kappa)$, $\tau'(x) > 0$, $\tau(x) \to \infty$ at $x = x_\tau$, where $q(x_\tau) = \kappa$
  - Recall that $q(x) = \left[1 + x^\gamma\right]^{-1/\gamma}$ with $\gamma > 0$
  - In a tighter market, visits are less likely to be successful, so households devote more resources to shopping $\rightsquigarrow$ larger shopping wedge

PRICE NORM

- Buyers and sellers are happy to trade:
  - If a seller does not sell her service, she remains idle
  - If a buyer does not buy the service, she has to incur the shopping cost again
  - So both buyers and sellers enjoy surplus from trade

$\rightsquigarrow$ Prices are determined in situation of bilateral monopoly

$\rightsquigarrow$ Price p is determined by price norm

$\rightsquigarrow$ Price norm is a custom about how to share surplus fairly

HOUSEHOLD PROBLEM

STATEMENT OF THE PROBLEM

- Choose c_i, m_i to maximize utility function:

$$\mathcal{U}\left(c_i, \frac{m_i}{p}\right) = \frac{\chi}{1+\chi} c_i^{\frac{\epsilon-1}{\epsilon}} + \frac{1}{1+\chi}\left(\frac{m_i}{p}\right)^{\frac{\epsilon-1}{\epsilon}}$$

- Subject to budget constraint:

$$p \cdot [1 + \tau(x)] \cdot c_i + m_i = p \cdot f(x) \cdot k_i + \mu_i$$

- Taking as given market tightness, x, and price of services, p
- Key novelties of slackish model:
  - Selling probability $f(x) < 1$: difficulty in finding buyers
  - Shopping wedge $\tau(x) > 0$: difficulty in finding sellers
  - Smooth functions of tightness instead of kinky regimes in Barro, Grossman (1971)

- Substitute budget constraint into utility:

$$\max_{c_i} \frac{\chi}{1+\chi} c_i^{\frac{\epsilon-1}{\epsilon}} + \frac{1}{1+\chi}\left[f(x)\cdot k_i + \frac{\mu_i}{p} - [1+\tau(x)]c_i \right]^{\frac{\epsilon-1}{\epsilon}}$$

- This is a concave maximization problem. The first-order condition gives a necessary and sufficient condition to find the global maximum of the problem:

$$\begin{aligned}
\frac{\epsilon-1}{\epsilon}\cdot\frac{\chi}{1+\chi}\cdot c_i^{-1/\epsilon} &= \frac{1}{1+\chi}\cdot\frac{\epsilon-1}{\epsilon}\left[1+\tau(x)\right]\left[f(x)\cdot k_i + \frac{\mu_i}{p} - [1+\tau(x)]c_i \right]^{-1/\epsilon} \\
c_i &= \left[\frac{\chi}{1+\tau(x)}\right]^{\epsilon}\left[f(x)\cdot k_i + \frac{\mu_i}{p} - [1+\tau(x)]c_i \right] \\
\left[1+\chi^{\epsilon}[1+\tau(x)]^{1-\epsilon}\right] c_i &= \chi^{\epsilon}[1+\tau(x)]^{-\epsilon}\left[f(x)\cdot k_i + \frac{\mu_i}{p} \right] \\
c_i &= \frac{\chi^{\epsilon}[1+\tau(x)]^{-\epsilon}}{1+\chi^{\epsilon}[1+\tau(x)]^{1-\epsilon}}\left[f(x)\cdot k_i + \frac{\mu_i}{p} \right].
\end{aligned}$$

OPTIMAL PURCHASES OF SERVICES

- Number of services that the household will actually purchase:

$$y_i = [1+\tau(x)]c_i = \frac{\chi^{\epsilon}[1+\tau(x)]^{1-\epsilon}}{1+\chi^{\epsilon}[1+\tau(x)]^{1-\epsilon}}\left[f(x)\cdot k_i + \frac{\mu_i}{p}\right]$$

- So household i spends a fraction $\phi(x) \in (0,1)$ of initial real wealth + real income on services:

$$y_i = \phi(x)\left[f(x)\cdot k_i + \frac{\mu_i}{p}\right]$$

$$\phi(x) = \frac{\chi^{\epsilon}[1+\tau(x)]^{1-\epsilon}}{1+\chi^{\epsilon}[1+\tau(x)]^{1-\epsilon}}$$

- Household i saves a fraction $1-\phi(x) \in (0,1)$ of initial real wealth + real income:

$$\frac{m_i}{p} = f(x)\cdot k_i + \frac{\mu_i}{p} - y_i = [1-\phi(x)]\left[f(x)\cdot k_i + \frac{\mu_i}{p}\right]$$

MARGINAL PROPENSITY TO SPEND

$$\phi(x) = \frac{\chi^{\epsilon}[1+\tau(x)]^{1-\epsilon}}{1+\chi^{\epsilon}[1+\tau(x)]^{1-\epsilon}}$$

- $\phi(x) \in (0,1)$: marginal propensity to spend out of total, post-income wealth
- $1 - \phi(x) \in (0,1)$: marginal propensity to save out of total, post-income wealth
- $\phi(x)$ is decreasing in x: marginal propensity to spend is lower in a tight economy
  - Because $\tau(x)$ is increasing in x
  - And $\epsilon > 1$ so $1 - \epsilon < 0$
  - And $z \mapsto z/(1+z)$ is increasing in z
- In tight economy, buying becomes more complicated as visits are less likely to be successful and a larger share of spending is devoted to shopping $\rightsquigarrow$ spending is less appealing

SUMMARY OF HOUSEHOLD'S BEHAVIOR

- Spending: $y_i = \phi(x)\left[f(x) \cdot k_i + \mu_i/p\right]$
- Saving: $m_i/p = [1 - \phi(x)]\left[f(x) \cdot k_i + \mu_i/p\right]$
- Consumption of services: $c_i = y_i/[1 + \tau(x)]$
- Shopping visits: $v_i = y_i/q(x)$
- Shopping services: $\kappa v_i = y_i - c_i = \tau(x)c_i$

AGGREGATE SUPPLY

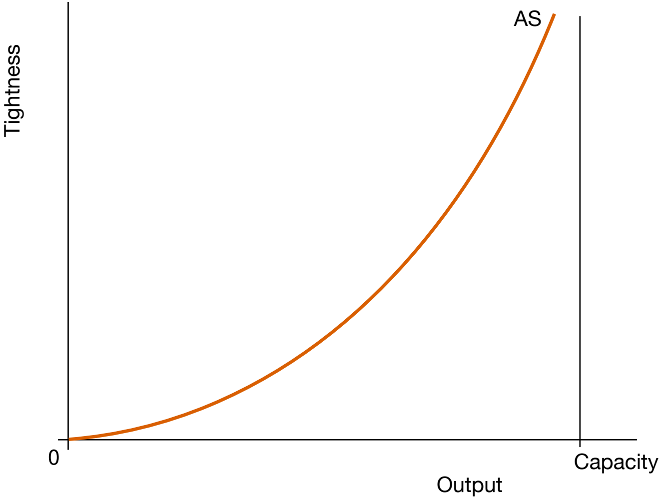
CONSTRUCTING THE AS CURVE

- Household i sells $y_i = f(x)k_i$
- All households sell $y = \sum y_i = f(x) \sum k_i = f(x)k$
- AS curve gives the number of services sold at tightness x:

$$y^s(x) = f(x) \cdot k$$

- AS curve has the same properties as the selling probability $f(x)$:
  - No sales without buyers: $y^s(0) = 0$
  - More sales in tighter markets: $d\, y^s/dx > 0$
  - AS curve is concave: $d^2\, y^s/dx^2 < 0$
  - All capacity is used with infinitely many buyers: $\lim_{x\to\infty} y^s(x) = k$
  - Recall that $f(x) = \left[1 + x^{-\gamma}\right]^{-1/\gamma}$ with $\gamma > 0$

PLOTTING THE AS CURVE



- Market diagram features tightness x on y-axis, not price p
- Tightness x is the central variable of the model:
  - Tightness determines all other variables
  - Tightness responds to shocks, not prices

AGGREGATE DEMAND

AGGREGATING HOUSEHOLDS' PURCHASES

- Household i purchases $y_i = \phi(x)[f(x)k_i + \mu_i/p]$
- All households purchase:

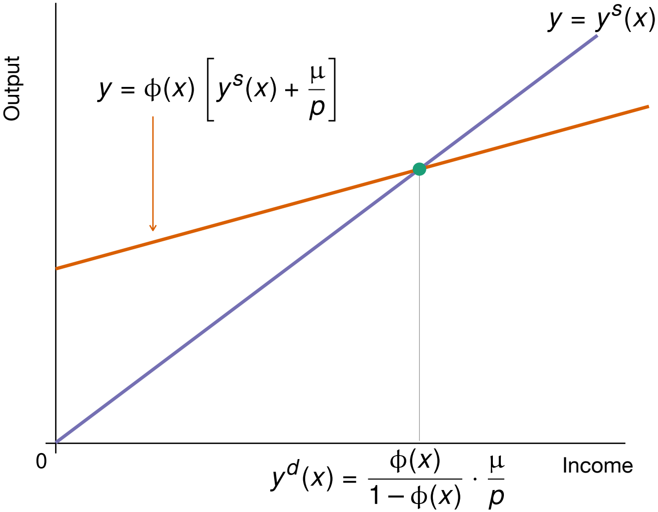
$$y = \sum y_i = \phi(x)\left[f(x)\left(\sum k_i\right) + \left(\sum \mu_i\right)/p\right] = \phi(x)\left[f(x)k + \mu/p\right]$$

- Aggregate quantity of services demanded:

$$y = \phi(x)\left[y^s(x) + \mu/p\right]$$

- Deviation from Say's Law because $\phi(x) < 1 \rightsquigarrow$ supply creates less than its own demand
- Nondegenerate aggregate demand because $\phi < 1$, so because $\chi < \infty$
  - Because people value not only consumption but also wealth

CONSTRUCTING THE AD CURVE USING A KEYNESIAN CROSS



- In the aggregate, income = spending = real output = y
- Income given by matching process: $y = y^s(x)$
- Spending chosen by households: $y = \phi(x)\left[y^s(x) + \mu / p\right]$
- AD curve is aggregate spending chosen by households given that aggregate income = aggregate spending

EXPRESSION FOR THE AD CURVE

- AD curve is given by:

$$y^d(x) = \frac{\phi(x)}{1 - \phi(x)} \cdot \frac{\mu}{p}$$

- But the marginal propensity to spend satisfies:

$$\phi(x) = \frac{\chi^{\epsilon}[1 + \tau(x)]^{1-\epsilon}}{1 + \chi^{\epsilon}[1 + \tau(x)]^{1-\epsilon}}$$

$$1 - \phi(x) = \frac{1}{1 + \chi^{\epsilon}[1 + \tau(x)]^{1-\epsilon}}$$

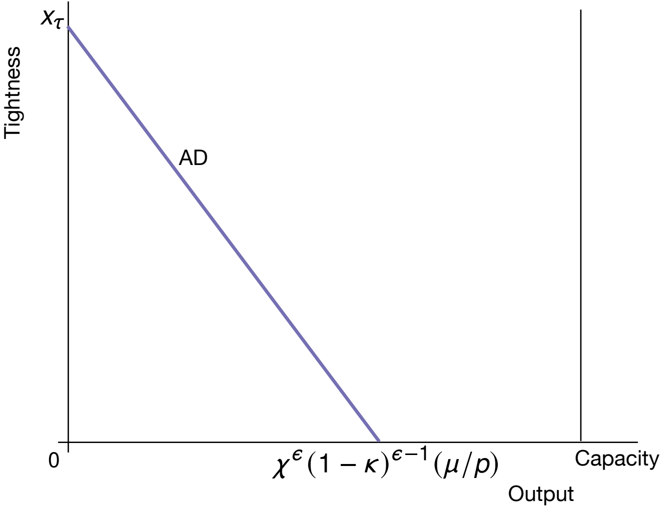
$$\frac{\phi(x)}{1 - \phi(x)} = \chi^{\epsilon}[1 + \tau(x)]^{1-\epsilon} = \frac{\chi^{\epsilon}}{[1 + \tau(x)]^{\epsilon-1}}$$

PROPERTIES OF THE AD CURVE

$$y^d(x) = \frac{\chi^\epsilon}{[1+\tau(x)]^{\epsilon-1}} \cdot \frac{\mu}{p}.$$

- χ: preference for consumption over saving
- $\tau(x)$: cost of shopping, from congestion
- μ/p: aggregate real wealth
- No spending in very tight economy: $y^d(x_\tau) = 0$, because $\tau(x_\tau) = 0$
- Less spending in tighter economy: $y^d(x)$ is decreasing in x
  - because $\tau(x)$ is increasing in x, and $\epsilon > 1$
- Maximum spending at 0 tightness: $y^d(0) = \chi^\epsilon(1-\kappa)^{\epsilon-1}(\mu/p)$, because $\tau(0) = \kappa/(1-\kappa)$

PLOTTING THE AD CURVE



- Market diagram features tightness x on y-axis, not price p
- But AD curve is also downward sloping in quantity-price diagram
- Same AD concept as IS curve in IS-LM $\rightsquigarrow$ microfoundation for IS

SOLUTION OF THE MODEL

TWO KEY RELATIONSHIPS

- Output = real income is given by the matching function:

$$y = y^s(x)$$

- Output = real spending is given by households' utility-maximizing decisions:

$$y = \phi(x)\left[y^s(x) + \frac{\mu}{p}\right]$$

- System of two equations and two variables: y, x

REWRITING THE SYSTEM WITH AS AND AD CURVES

- If we substitute the first equation into the second one, we obtain:

$$\begin{cases} y = y^s(x) \\ y = \phi(x)\left[y + \frac{\mu}{p}\right] \end{cases}$$

- This just gives the following system:

$$\begin{cases} y = y^s(x) \\ y = \frac{\phi(x)}{1-\phi(x)} \cdot \frac{\mu}{p} \end{cases}$$

- Which we can re-express with AS and AD curves:

$$\begin{cases} y = y^s(x) \\ y = y^d(x) \end{cases}$$

SOLUTION OF THE MODEL

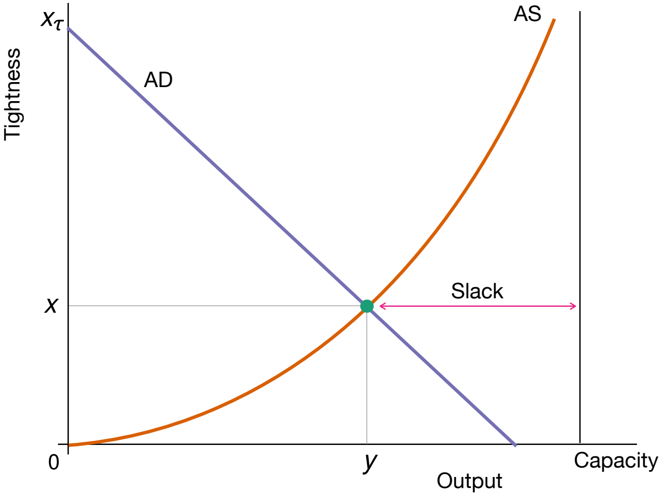
- Tightness x that solves the model is given by:

$$y^s(x) = y^d(x)$$

- Then output can be read from the AS or AD curves
- $y^s(x)$ is increasing from 0 to k as x is increasing from 0 to ∞
- $y^d(x)$ is decreasing from $\chi^\epsilon(1-\kappa)^{\epsilon-1}(\mu/p)$ to 0 as x is increasing from 0 to x_τ

$\rightsquigarrow$ Model always admits a unique solution, $x \in (0, x_\tau)$

GRAPHICAL REPRESENTATION OF THE SOLUTION



- Market diagram features tightness x on y-axis, not price p
- Intersection of AD and AS curves gives y, x that solve the model
- All other variables can be computed from x
- Model all features slack = $1 - f(x)$
- Slack is share of idle capacity

COMPUTING AGGREGATE VARIABLES FROM TIGHTNESS

- Aggregate output: $y = y^s(x) = y^d(x)$
- Aggregate consumption: $c = y/[1 + \tau(x)]$
- Rate of slack = rate of idleness = $1 - f(x)$
- Aggregate wealth holdings: $m = \mu$ (Walras's Law)
- Aggregate shopping visits: $v = y/q(x)$
- Price p is given by price norm, which could depend on x

COMPUTING INDIVIDUAL VARIABLES FROM TIGHTNESS

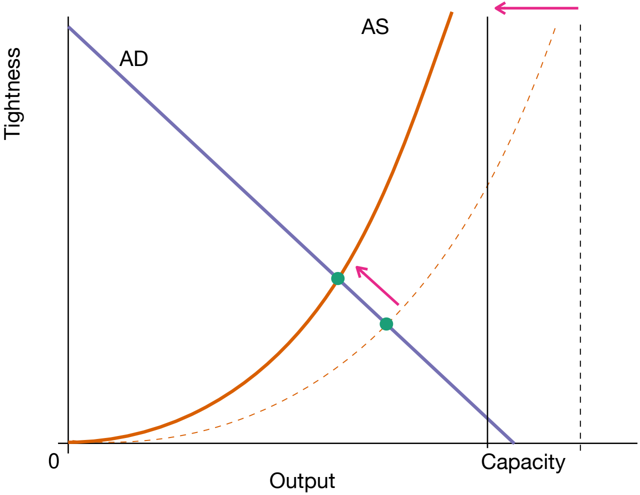
- Individual real income: $f(x)k_i$
- Individual real spending: $y_i = \phi(x)[f(x)k_i + \mu_i/p]$
- Individual rate of idleness: $1 - f(x)$
- Individual wealth holdings: $m_i/p = [1 - \phi(x)][f(x)k_i + \mu_i/p]$
- Individual shopping visits: $v_i = y_i/q(x)$
- Individual consumption: $c_i = y_i/[1 + \tau(x)]$

HOW MUCH RATIONALITY DOES THE MODEL ASSUME?

- Households maximize utility subject to budget constraint $\rightsquigarrow$ do the best they can given their income and wealth
- Households anticipate the price of services $\rightsquigarrow$ reasonable since prices are given by price norms which are by definition well understood
  - Realized prices can differ from the norm as long as they remain within the bargaining set, and that such deviations are unanticipated
- Households anticipate the market tightness $\rightsquigarrow$ this is difficult, but tightness might be announced by a statistical agency whose goal is to make correct predictions
  - The only announced tightness that can be realized is the model solution
  - See lecture video for more details

COMPARATIVE STATICS WITH FIXED PRICES

AS SHOCKS

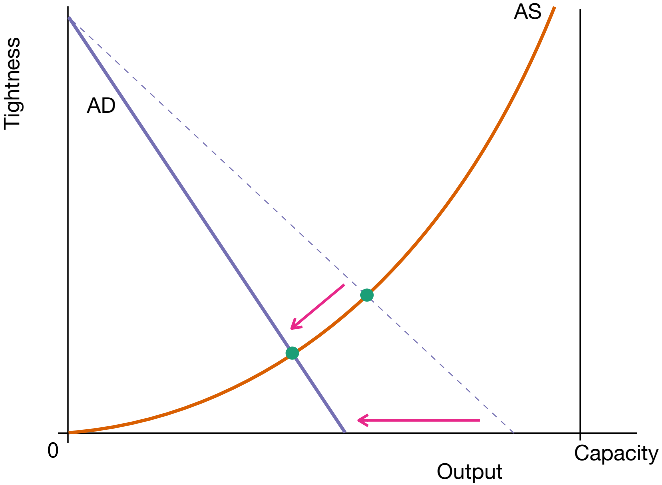


- $y^s(x) = f(x)k$
- Change in capacity k: labor force participation, immigration
- Negative correlation between output and tightness
- Movement along the AD curve

IMPACT OF NEGATIVE AS SHOCK

- Consider a negative AS shock: capacity $k \downarrow$
- Tightness $x \uparrow$
- Output $y \downarrow$
- Slack/idleness $1 - f(x) \downarrow$
- Buying probability $q(x) \downarrow$, and shopping wedge $\tau(x) \uparrow$
- Selling probability $f(x) \uparrow$ (also measured productivity $\downarrow$)
- Consumption $c = y/[1 + \tau(x)] \downarrow$
- Negative AS shock reduces output and makes it harder to buy goods:
  - Describes the lockdown period of the pandemic well
  - But rare in general since slack and output comove negatively (Okun's law)

AD SHOCKS



- $y^d(x) = \chi^{\epsilon}(\mu/p)[1 + \tau(x)]^{1-\epsilon}$
- Change in desire to consume χ or wealth endowment μ
- Positive correlation between output and tightness
- Movement along the AS curve

IMPACT OF NEGATIVE AD SHOCK

- Consider a negative AD shock: desire to consume $\chi \downarrow$ or wealth endowment $\mu \downarrow$
- Tightness $x \downarrow$
- Output $y \downarrow$
- Slack/idleness $1 - f(x) \uparrow$
- Buying probability $q(x) \uparrow$, and shopping wedge $\tau(x) \downarrow$
- Selling probability $f(x) \downarrow$ (also measured productivity $\downarrow$)
- Negative AD shock reduces output and makes it harder to sell goods:
  - Most common shock since slack and output comove negatively (Okun's law)

GENERALIZATION TO RIGID PRICES

BREAKING DOWN THE COMPARATIVE STATICS

- Tightness is given by $y^s(x) = y^d(x, p)$:

$$f(x)k = \chi^{\epsilon}[1 + \tau(x)]^{1-\epsilon}\frac{\mu}{p}$$

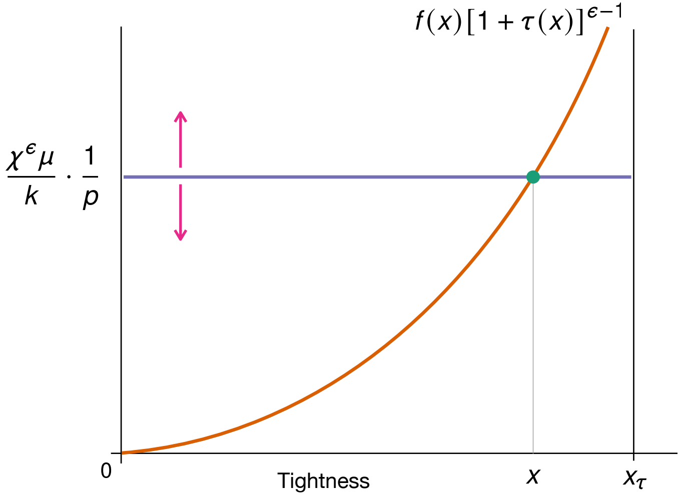
- Collect all tightness terms to isolate the source of comparative statics:

$$f(x)[1 + \tau(x)]^{\epsilon-1} = \frac{\chi^{\epsilon}\mu}{k} \cdot \frac{1}{p}$$

- Left-hand side is strictly increasing from 0 to ∞ when x increases from 0 to $x_\tau \rightsquigarrow$ equation admits a unique solution (the solution of the model)
- What determines the response of tightness is the block:

$$\frac{\chi^{\epsilon}\mu}{k} \cdot \frac{1}{p}$$

ILLUSTRATION OF COMPARATIVE STATICS



- The intersection of the two curves is the model solution
- The term $(\chi^\epsilon \mu)/(k\,p)$ varies in comparative statics
- Any price norm p that does not completely absorb variations in $(\chi^\epsilon \mu)/k$ yields the same results

RIGID PRICE NORM

- The comparative-static results remain the same when the price is not fixed but partially rigid:

$$p = p_0 \cdot \left[\frac{\chi^{\epsilon}\mu}{k}\right]^{1-\rho}$$

- $p_0 > 0$ governs the price level
- $\rho \in (0, 1)$ governs the rigidity of prices:
  - $\rho = 0$: flexible price (surplus-sharing price as special case)
  - $\rho = 1$: fixed price
- χ, μ: AD parameters, tend to raise price
- k: AS parameter, tend to lower price

COMPARATIVE STATICS WITH PARTIALLY RIGID PRICES

- The $y^s(x) = y^d(x, p)$ equation becomes:

$$f(x)[1+\tau(x)]^{\epsilon-1} = \left[\frac{\chi^{\epsilon}\mu}{k}\right]^{\rho} \cdot \frac{1}{p_0}$$

- Same implicit definition of x except for exponent $\rho > 0$
- Since $\rho > 0$, the right-hand side moves in the same direction as with fixed price
- All comparative statics remain the same
- But size of effects are attenuated $\rightsquigarrow$ elasticity of x with respect to χ, μ, k is ρ the elasticity under fixed price
  - $\rightsquigarrow$ Effects vanish as price becomes flexible ($\rho \to 0$)
- There is nothing special about fixed prices $\rightsquigarrow$ flexible prices are special