

PROBLEM SET ON MALTHUSIAN MODEL OF GROWTH

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Problem 1

Consider a Malthusian model with production function $Y = 5 \times L^{3/4}$. The variable Y is output and the variable L is working population. People live two periods. Each family has one parent and N children. Each child consumes 2 unit of food, and a parent consumes C . Each parent maximizes her utility $U = N^{1/4} \times C^{3/4}$.

- A) Compute output per worker y .
- B) Is output per worker, y , increasing or decreasing in the size of the working population, L ?
- C) Give the budget constraint of a family with N children, and explain.
- D) Using the budget constraint, give the number of children N that a parent with consumption C can afford.
- E) Using your answer to D), compute the consumption C that maximizes a parent's utility U subject to her budget constraint.
- F) Compute the optimal number of children N for a parent. (That is, compute the number of children consistent with the consumption derived in E) and the parent's budget constraint.)
- G) Combining your previous answers, express the working population at time $t+1$, $L(t+1)$, as a function of the working population at time t , $L(t)$.
- H) How many children does each parent have in steady state? Explain.
- I) What is the steady-state level of the working population, L^* ?
- J) Imagine that a new farming technique is discovered such that the production function becomes $Y = 7 \times L^{3/4}$. What is the new value of L^* ? Has L^* increased or decreased? Explain.

- K) Draw the usual population diagram. Give the equation for each curve and show the steady state.
- L) Use the diagram to show the effect of the new farming technique. Explain.
- M) Plot the evolution of population over time: in the old steady state, right after the discovery of the farming technique, and in the new steady state. Explain.
- N) Using your previous answers, express the output per worker at time $t+1$, $y(t+1)$, as a function of the output per worker at time t , $y(t)$.
- O) What is the steady-state level of output per worker, y^* ?
- P) What is the new value of y^* after the new farming technique is discovered? Has y^* increased or decreased? Explain.
- Q) Draw the usual output-per-worker diagram. Describe each curve and show the steady state.
- R) Use the diagram to show the effect of the new farming technique.
- S) Plot the evolution of output per workers over time: in the old steady state, right after the discovery of the farming technique, and in the new steady state. Explain.
- T) Starting from the steady state, assume that a disease kills 20% of the population. Using the diagrams from questions K) and Q), describe what happens to population and output per worker over time. Explain.