Convergence to the Beveridge Curve

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Law of motion of unemployment rate u:

$$\dot{u}(t) = -[\lambda + f]\, u(t) + \lambda$$

job-finding rate

job-separation rate

Critical point: $\bar{u}$ such that $\dot{u} = 0$

$$\dot{u} = 0 \Rightarrow -(\lambda + f)\,\bar{u} + \lambda = 0$$

$$\Rightarrow \boxed{\bar{u} = \frac{\lambda}{\lambda + f}}$$

$$\Rightarrow \boxed{u(\theta) = \frac{\lambda}{\lambda + f(\theta)}}$$

Beveridge curve.

$$\lambda + f(\theta) = \frac{\lambda}{u}$$

$$f(\theta) = \frac{\lambda - \lambda u}{u}$$

$$\mu\, \theta^{1-\eta} = \frac{\lambda - \lambda u}{u}$$

$$\theta = \left[\frac{\lambda - \lambda u}{\mu \cdot u}\right]^{\frac{1}{1-\eta}} = \frac{v}{u}$$

$$v = \left[\frac{\lambda - \lambda u}{\mu \quad u} \cdot u^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

$$\boxed{v = \left[\frac{\lambda - \lambda u}{\mu \, u^{\eta}} \right]^{\frac{1}{1-\eta}}}$$

Beveridge curve. $v(u)$ w/ $\frac{dv}{du} < 0$

Convergence of $u(t)$ to $u(\theta)$

$$\mathring{u}(t) + (\lambda + f)\, u(t) = \lambda$$

initial gap, shrinking

Solution,

$$\boxed{[u(t) - \underbrace{u(\theta)}_{\text{gap}}] = [u(0) - u(\theta)]\, e^{-(\lambda+f)\,t}}$$

$$\Rightarrow \quad u(t) = u(\theta) + [u(0) - u(\theta)]\, e^{-(\lambda+f)t}$$

$$\mathring{u}(t) = -(\lambda+f)\,[u(0) - u(\theta)]\, e^{-(\lambda+f)t}$$

$$\mathring{u}(t) = -(\lambda+f) \times [u(t) - u(\theta)]$$

$$\mathring{u}(t) + (\lambda+f)\, u(t) = (\lambda+f)\, u(\theta) = \lambda$$

differential equation is satisfied

+ satisfy initial condition at $t=0$

Interpretation: Gap b/w $u(t)$ & Beveridge curve shrinks at rate $(\lambda+f)$

$\Rightarrow$ $\lambda + f$ is speed at which unemployment converges to Beveridge curve.

In US:
- $\lambda \approx 3\%$ per month
- $f \approx 59\%$ per month

$\Rightarrow$ $\lambda + f \approx 62\%$ per month

Half time: time it takes for unemployment rate to cover 1/2 distance to Beveridge curve.

$$\frac{\ln(2)}{\lambda + f} = \frac{0.69}{0.62} \approx 1.2 \text{ month.}$$

In a quarter, $(1/2)^3 = 1/8$ of initial $\approx 10\%$ distance w/ Beveridge curve is left.

$\Rightarrow$ Convergence to Beveridge curve is very fast b/c λ, f are large

$\Rightarrow$ Unemployment rate always on Beveridge curve

$$\boxed{u = u(\theta) = \frac{\lambda}{\lambda + f(\theta)}}$$