# Matching with Long-Term Employment Relationships

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**Notation.**

- $h > 0$ : labor force
- $\ell(t)$ : employment
- $v(t)$ : vacancies
- $m(t)$ : # new matches at $t$
- $f(t)$ : job-finding rate
- $q(t)$ : recruiting rate
- $\lambda > 0$ : job-separation rate

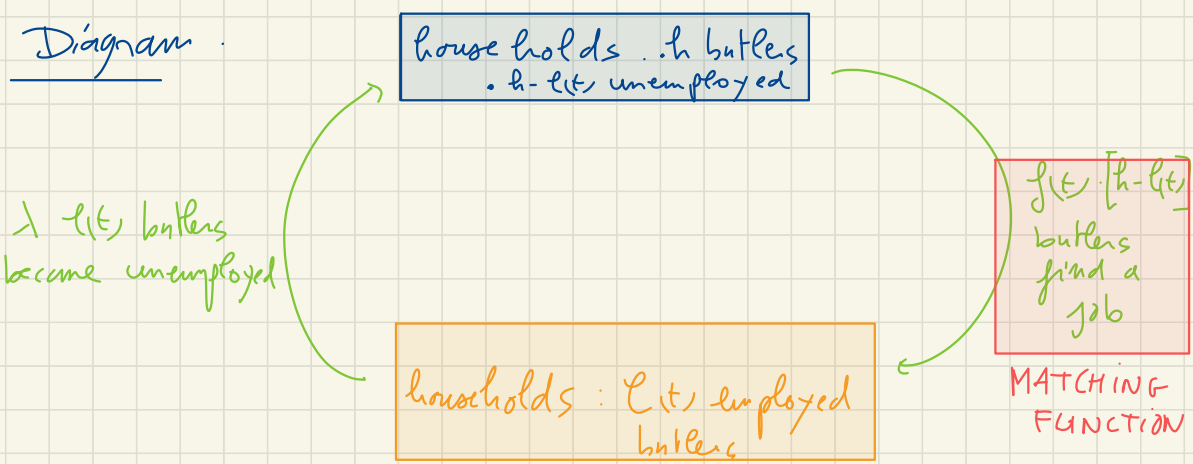
**Framing of matching market.** market for butlers

→ butlers produce services consumed by households

→ each butler produces $a > 0$ service per unit time

→ butlers are in long-term employment relationships w/ households

**Diagram.**



Matching function — Cobb-Douglas matching function

2 arguments: - $v(t)$: # vacancies posted by households

- $h - l(t)$ # unemployed workers

# matches per unit time:

$$m(t) = \mu \, v(t)^{1-\eta} \, [h - l(t)]^{\eta}$$

$\mu > 0$: matching efficacy

$\eta$: matching elasticity (elasticity of matching function wrt unemployment)

$$\eta = d\ln m(t) / d\ln(h - l(t))$$

Matching rates:

- Market tightness: $\theta(t) = \frac{v(t)}{h - l(t)}$

- Job-finding rate: $f(t) = \frac{m(t)}{h - l(t)} = \mu \cdot \theta(t)^{1-\eta}$ ($f(\theta(t))$)

- Recruiting rate: $q(t) = \frac{m(t)}{v(t)} = \mu \cdot \theta(t)^{-\eta}$ ($q(\theta(t))$)