Solving the Two-Market Model

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Solve the two-market model (x, θ) such that

$$\begin{cases} y^d(x) = y^s(x, \theta) & (AD = AS) \\ \ell^d(x, \theta) = \ell^s(\theta) & (LD = LS) \end{cases}$$

- $y^d(x) = y^s(x, \theta)$

$$\Leftrightarrow \frac{X^{\varepsilon}}{[1+\tau(x)]^{\varepsilon-1}} \cdot \frac{\mu}{p} = f(x) \cdot a \cdot \left[\frac{\hat{f}(\theta)\, h}{1+\hat{\tau}(\theta)}\right]^{\alpha}$$

$$\Leftrightarrow \boxed{f(x)\,[1+\tau(x)]^{\varepsilon-1} = \frac{X^{\varepsilon}\mu}{p\, a\, h^{\alpha}} \cdot \left[\frac{1+\hat{\tau}(\theta)}{\hat{f}(\theta)}\right]^{\alpha}} \quad (P)$$

- $\ell^d(x, \theta) = \ell^s(\theta)$

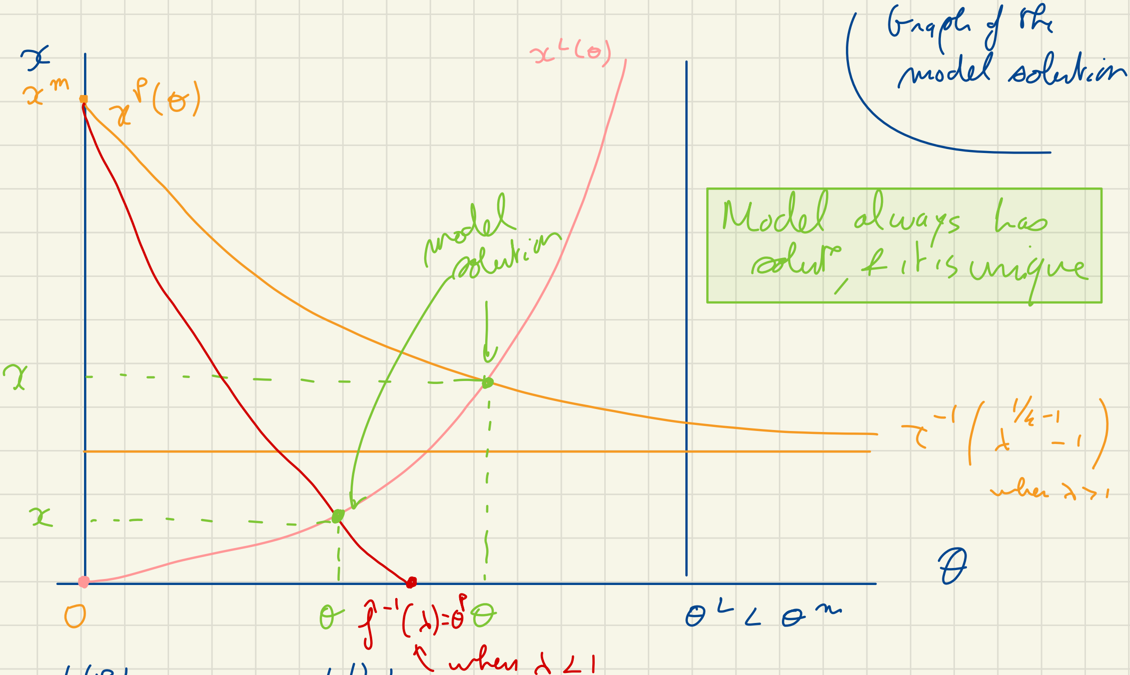
$$\Leftrightarrow \left[\frac{f(x)\, a\, \alpha}{w/p}\right]^{1/1-\alpha} \left[\frac{1}{1+\hat{\tau}(\theta)}\right]^{\alpha/1-\alpha} = \hat{f}(\theta)\, h$$

$$\Leftrightarrow \boxed{f(x) = \frac{w/p \cdot h^{1-\alpha}}{a\,\alpha} \hat{f}(\theta)^{1-\alpha} [1+\hat{\tau}(\theta)]^{\alpha}} \quad (L)$$

What do we learn from (L)

$$x = x^L(\theta) = f^{-1}\left(\underbrace{\frac{w/p}{a\,\alpha} h^{1-\alpha}\, \hat{f}(\theta)^{1-\alpha}\, [1+\hat{\tau}(\theta)]^{\alpha}}_{\text{at } \theta^L = 1}\right)$$

- x^L is strictly $\nearrow$ in θ
- $x^L(0) = f^{-1}(0) = 0$
- θ^L st $x^L(\theta^L) = f^{-1}(1) = +\infty$
  with $\theta^L < \theta^m$



$$\begin{cases}(P)\\(L)\end{cases} \iff \begin{cases}(L)\\(P)/(L)\end{cases}$$

$$(P)/(L) \iff [1+\tau(x)]^{\varepsilon-1} = \frac{X^{\varepsilon}\nu\,\cancel{\alpha}\,\alpha}{\cancel{\rho}\,\cancel{\alpha}\cdot\cancel{h^{\alpha}}\,\frac{w}{\cancel{\rho}}\,h^{\cancel{1-\alpha}}}\cdot\frac{1}{\bar{\rho}(\theta)}$$

$$\iff \tau(x) = \left[\frac{X^{\varepsilon}\nu\alpha}{w\cdot h}\cdot\frac{1}{\bar{\rho}(\theta)}\right]^{\frac{1}{\varepsilon-1}} - 1$$

$$\iff x = x^P(\theta) = \tau^{-1}\left[\left(\frac{X^{\varepsilon}\nu\alpha}{w\cdot h}\cdot\frac{1}{\bar{\rho}(\theta)}\right)^{\frac{1}{\varepsilon-1}} - 1\right]$$

- x^P is strictly ↓ in θ
- $x^P(0) = x^m$ b/c $\tau(x^m) = +\infty$
- Depending on whether $\left(\frac{X^{\varepsilon}\nu\alpha}{w\,h}\right)^{1/\varepsilon-1} - 1$ is >0 or <0.

I . $\left(\frac{X^{\varepsilon}\nu\alpha}{w \cdot h}\right)^{1/\varepsilon-1} - 1 > 0 \quad \lambda > 1$

Then $\lim_{\theta \to \infty} x^P(\theta) = x^P$ st

$\tau(x^P) = \lambda^{1/\varepsilon-1} - 1 \quad \alpha \quad \boxed{x^P = \tau^{-1}\left(\lambda^{1/\varepsilon-1} - 1\right)}$

II . $\lambda < 1$. There is θ^P such that

$$\left(\frac{X^{\varepsilon}\nu\alpha}{w \cdot h} \frac{1}{\hat{f}(\theta^P)}\right)^{1/\varepsilon-1} = 1$$

$$\Leftrightarrow \hat{f}(\theta^P) = \frac{X^{\varepsilon}\nu\alpha}{w \cdot h} = \lambda \quad \frac{X^{\varepsilon}\nu\alpha}{w \cdot h}$$

$$\Leftrightarrow \boxed{\theta^P = \hat{f}^{-1}(\lambda)}$$

Then $x^P(\theta^P) = \tau^{-1}(0) = 0$