# Firm's Problem

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Firms maximize profits by choosing # of producers $n$, taking as given market tightnesses $x$, $\theta$ and prices $p$, $w$.

( Equivalent: - firm chooses # employees $e$
[ b/c $e = [1+\hat{\tau}(\theta)]\, n$ ]
- firm chooses # vacancies $\hat{v}$
[ b/c $\hat{v} = e / \hat{q}(\theta)$ ] )

Profits: Revenue − Cost

= Revenue from sale of services − Wage bill

$$= p \cdot f(x) \cdot k - w \cdot e$$

price of service, selling proba, capacity, nominal wage, employees

$$= p \cdot f(x) \cdot a \cdot n^{\alpha} - w\,[1+\hat{\tau}(\theta)] \cdot n$$

technology, matching wedge, producers

function of n: concave in n ($\alpha < 1$)

standard concave maximization problem

$$\max_{n>0} \quad p \cdot a \cdot f(x)\, n^{\alpha} - w\,[1+\hat{\tau}(\theta)]\, n$$

↳ derivative. FOC

producer ↓ ; wage for recruiter to get producer

$$\underbrace{p \cdot a \cdot f(x)\, \alpha\, n^{\alpha-1}}_{\text{MR from 1 producer}} = \underbrace{w\,[1+\hat{\tau}(\theta)]}_{\text{MC of product}}$$

↳ sufficient condition for global maximum

MR from 1 producer: MPL × price × selling proba

MC of product: cost of 1 extra producer

$$\Leftrightarrow \quad n^{\alpha-1} = \frac{[1+\hat{\tau}(\theta)]}{a\,\alpha\, f(x)} \frac{w}{p}$$

$$\Leftrightarrow \quad \boxed{n = \left[\frac{a \cdot \alpha \cdot f(x)}{[1+\hat{\tau}(\theta)] \cdot (w/p)}\right]^{\frac{1}{1-\alpha}}}$$

Profit-maximizing # employees: $\ell = [1+\hat{\tau}(\theta)]\, n$

$$\ell = \left[\frac{a\,\alpha\, f(x)}{w/p}\right]^{1/1-\alpha} \cdot [1+\hat{\tau}(\theta)]^{\underbrace{1-1/1-\alpha}_{-\alpha/1-\alpha}}$$

$$\Leftrightarrow \quad \boxed{\ell = \left[\frac{a\,\alpha \cdot f(x)}{w/p}\right]^{1/1-\alpha} \cdot \left[\frac{1}{1+\hat{\tau}(\theta)}\right]^{\alpha/1-\alpha}}$$