Cobb-Douglas Matching Function

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$M = \#$ trades

$S = \#$ sellers

$B = \#$ buyers

$$\boxed{M = \omega \, S^{\eta} \cdot B^{1-\eta}}$$

- $m(0, B) = m(S, 0) = 0$
- $\dfrac{\partial m}{\partial S} > 0 \qquad \dfrac{\partial m}{\partial B} > 0$
- Constant returns to scale

$$m(\lambda S, \lambda B) = \omega \cdot (\lambda S)^{\eta} (\lambda B)^{1-\eta}$$

$$= \omega \underbrace{\lambda^{\eta} \lambda^{1-\eta}}_{\lambda} S^{\eta} B^{1-\eta}$$

$$= \lambda \, \omega \, S^{\eta} B^{1-\eta}$$

$$m(\lambda S, \lambda B) = \lambda \, m(S, B)$$

Cobb-Douglas function can be <u>calibrated</u>

ω : matching efficacy

η matching elasticity

(exponent on # of sellers

1 Trading probabilities are simple.

$$f(\theta) = \text{selling proba} = \frac{M}{S} = \omega\, S^{\eta-1}\, B^{1-\eta}$$

$$= \omega \left(\frac{B}{S}\right)^{1-\eta}$$

$$\boxed{f(\theta) = \omega\, \theta^{1-\eta}}$$

$$q(\theta) = \text{buying proba} = \frac{M}{B} = \omega\, S^{\eta}\, B^{-\eta}$$

$$q(\theta) = \omega \left(\frac{S}{B}\right)^{\eta}$$

$$\boxed{q(\theta) = \omega \cdot \theta^{-\eta}}$$

$$\boxed{\theta = B/S}$$

2. Cobb Douglas is realistic matching function (Petrongolo & Pissarides 2001)

$$\boxed{0.5 \leq \eta \leq 0.7}$$ → common to calibrate $\eta = 0.5$