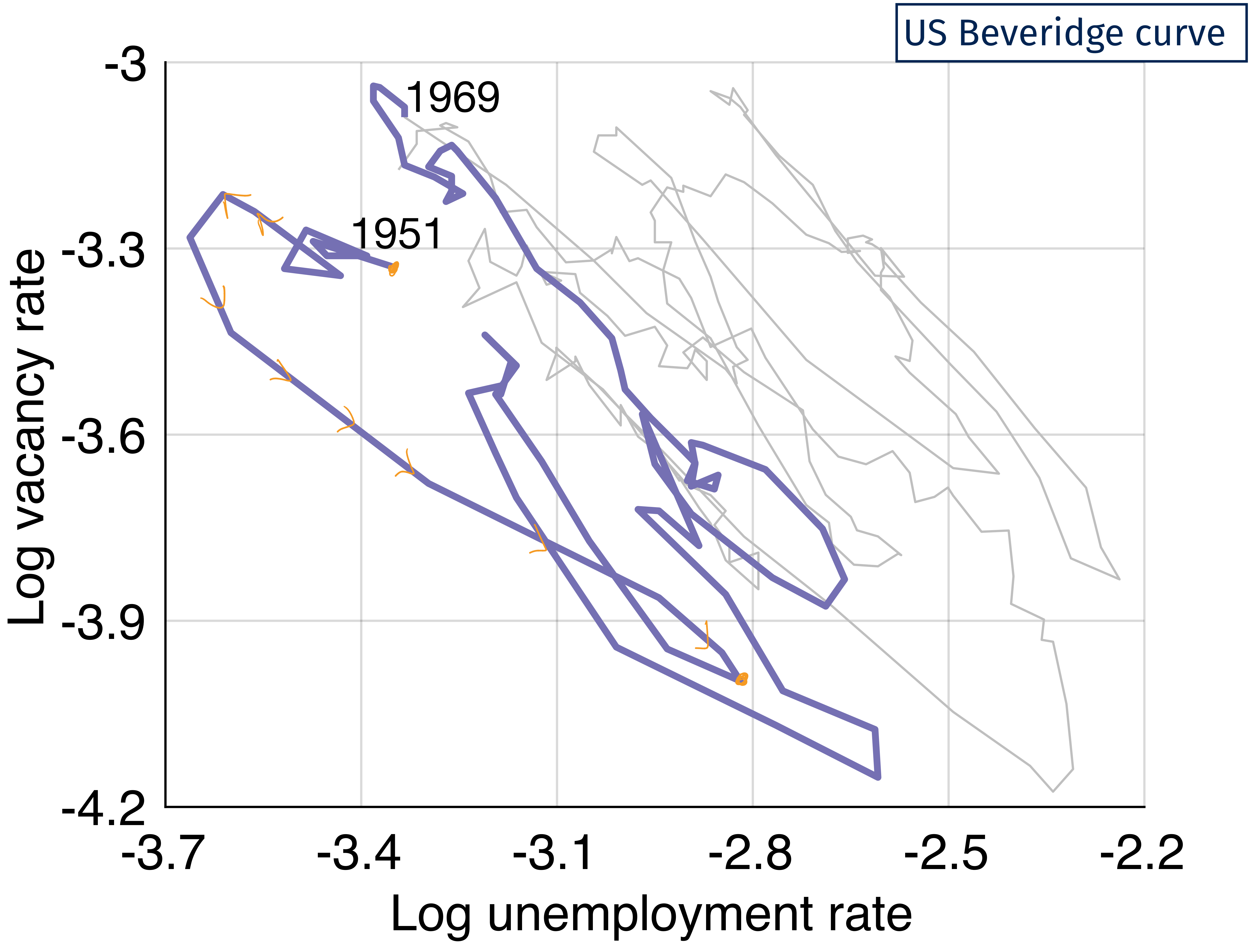
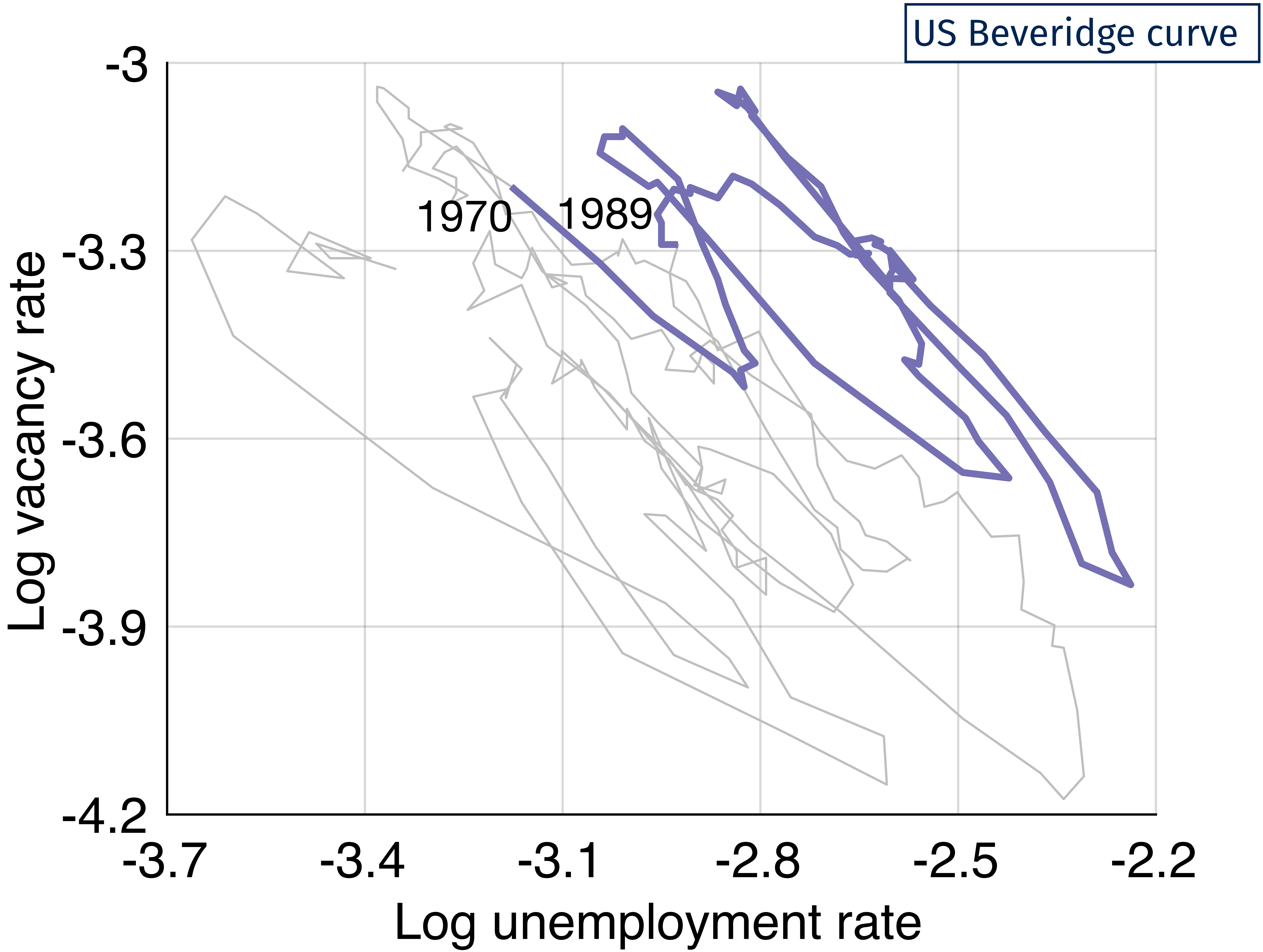
Wage Functions

Pascal Michaillat
https://pascalmichaillat.org/c1/

- average unemployment rate, 1948–2020: 5.8%
- unemployment goes up in recessions
- unemployment fluctuated between 2.5% and 15% in 1948–2020



US Beveridge curve
1969
1951
-3
-3.3
-3.6
-3.9
-4.2
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log vacancy rate
Log unemployment rate


US Beveridge curve
1970
1989
-3
-3.3
-3.6
-3.9
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-3.7
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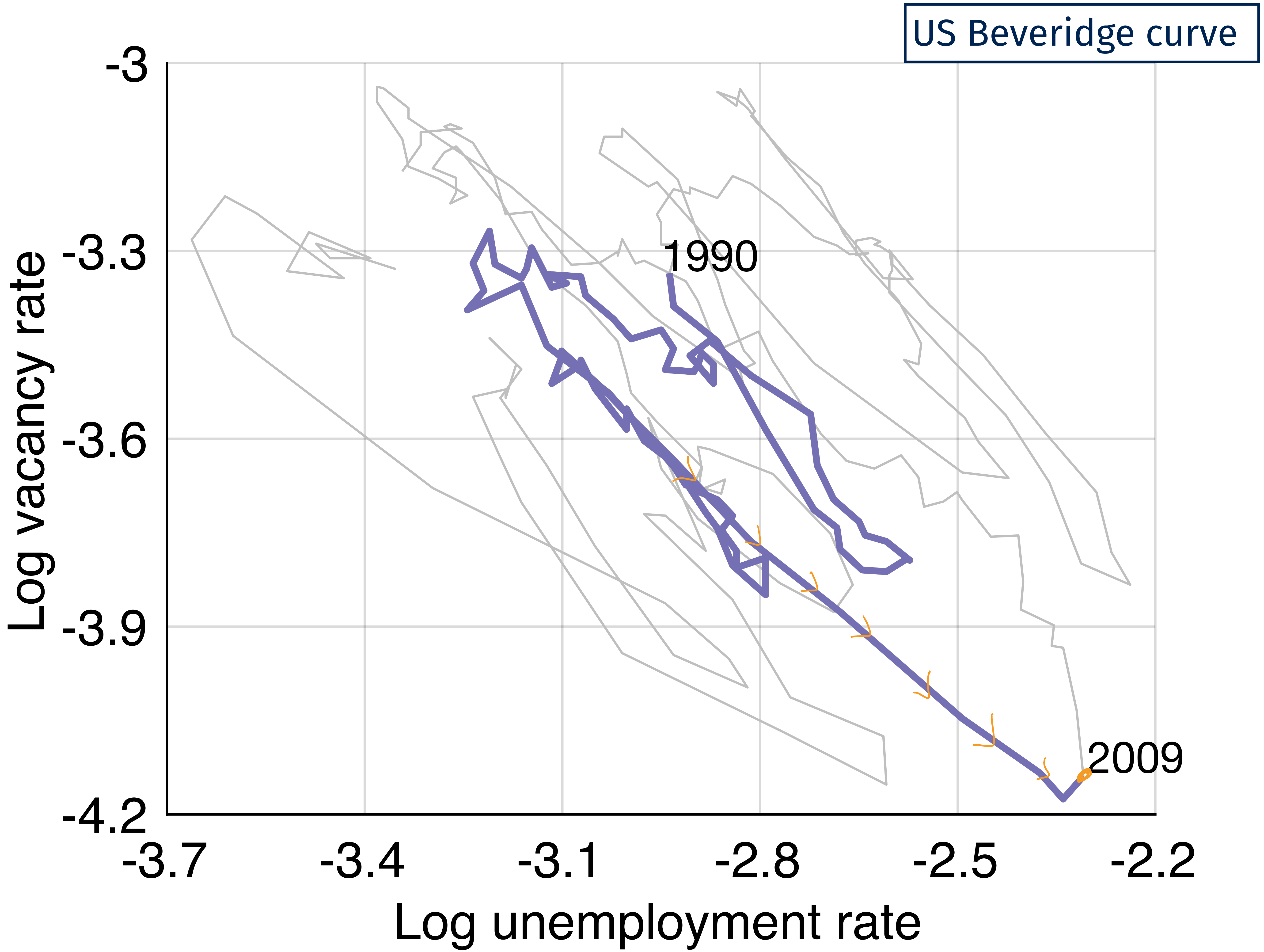

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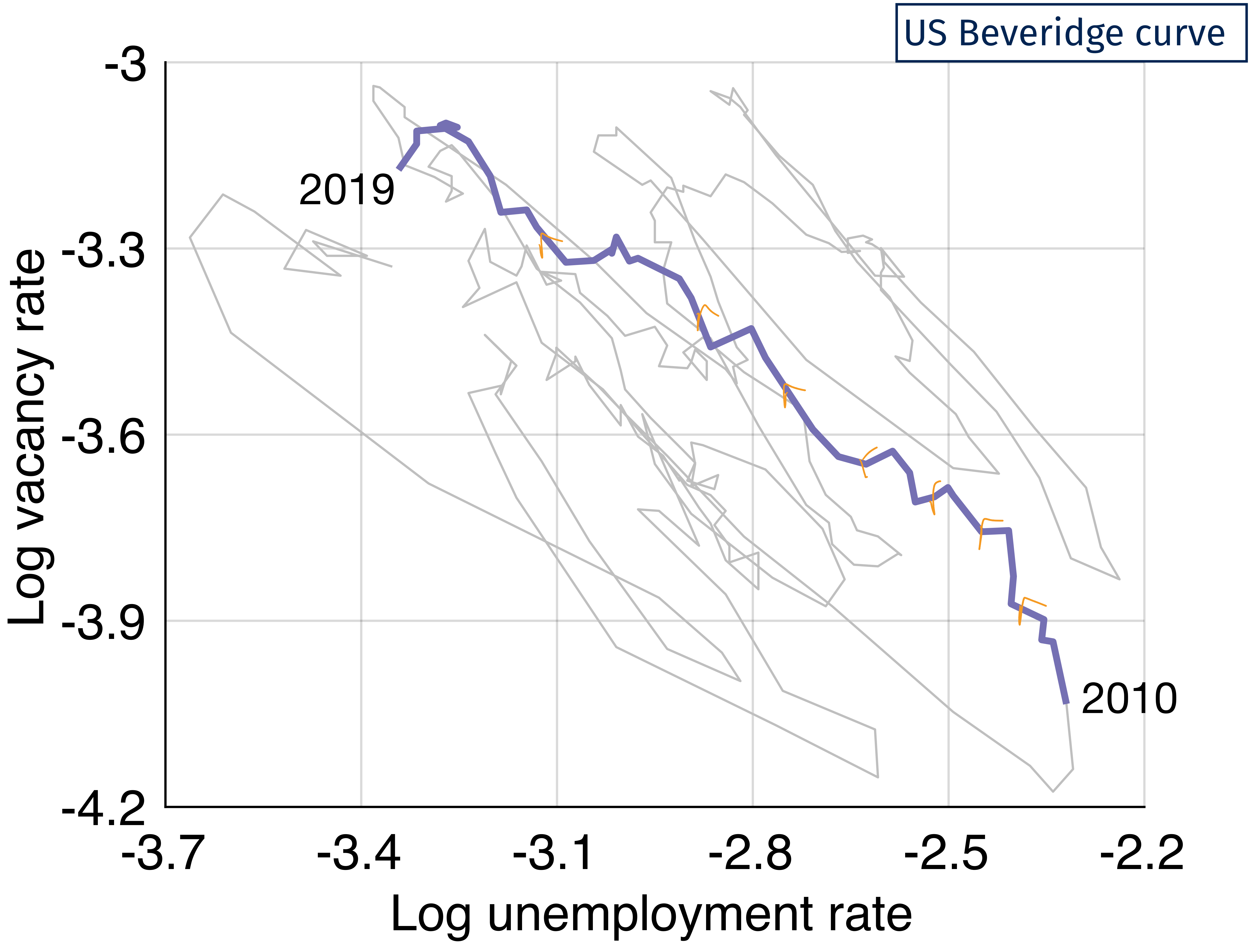

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Explaining unemployment fluctuations

- large fluctuations in unemployment : countercyclical
- negative correlation b/w unemployment rate & vacancy rate : Beveridge curve

Computing unemployment in the matching model :

labor market tightness given by equilibrium condition :

$$\boxed{L^s(\theta) = L^d(\theta)}$$

$$\frac{f(\theta)}{s + f(\theta)} \cdot H = \left[\frac{a \cdot \alpha}{w \, [1 + \tau(\theta)]^{\alpha}} \right]^{1/(1-\alpha)}$$

↳ defines implicitly the tightness θ.

↳ $u(\theta) = \dfrac{s}{s + f(\theta)}$

↳ $v(\theta) = \theta * u(\theta)$

Potential sources of unemployment fluctuations :

① a : productivity parameter → labor demand shock

② s : job-separation rate

③ H: size of labor force

labor supply shocks

Wage-setting in the matching model:

① Properties of wage W are key to determine business-cycle fluctuations in unemployment & vacancies.

② Wage W is specific to each worker-firm pair. (not a market wage)

→ pricing function describes wage W paid by firms to workers

③ there are many possible pricing function

→ workers & firms meet in a situation of bilateral monopoly (workers & firms have some bargaining power → difficult to find new match)

→ there are many possible prices in this situation (infinitely many prices, within a range).

↳ use evidence from real labor markets to specify pricing function.

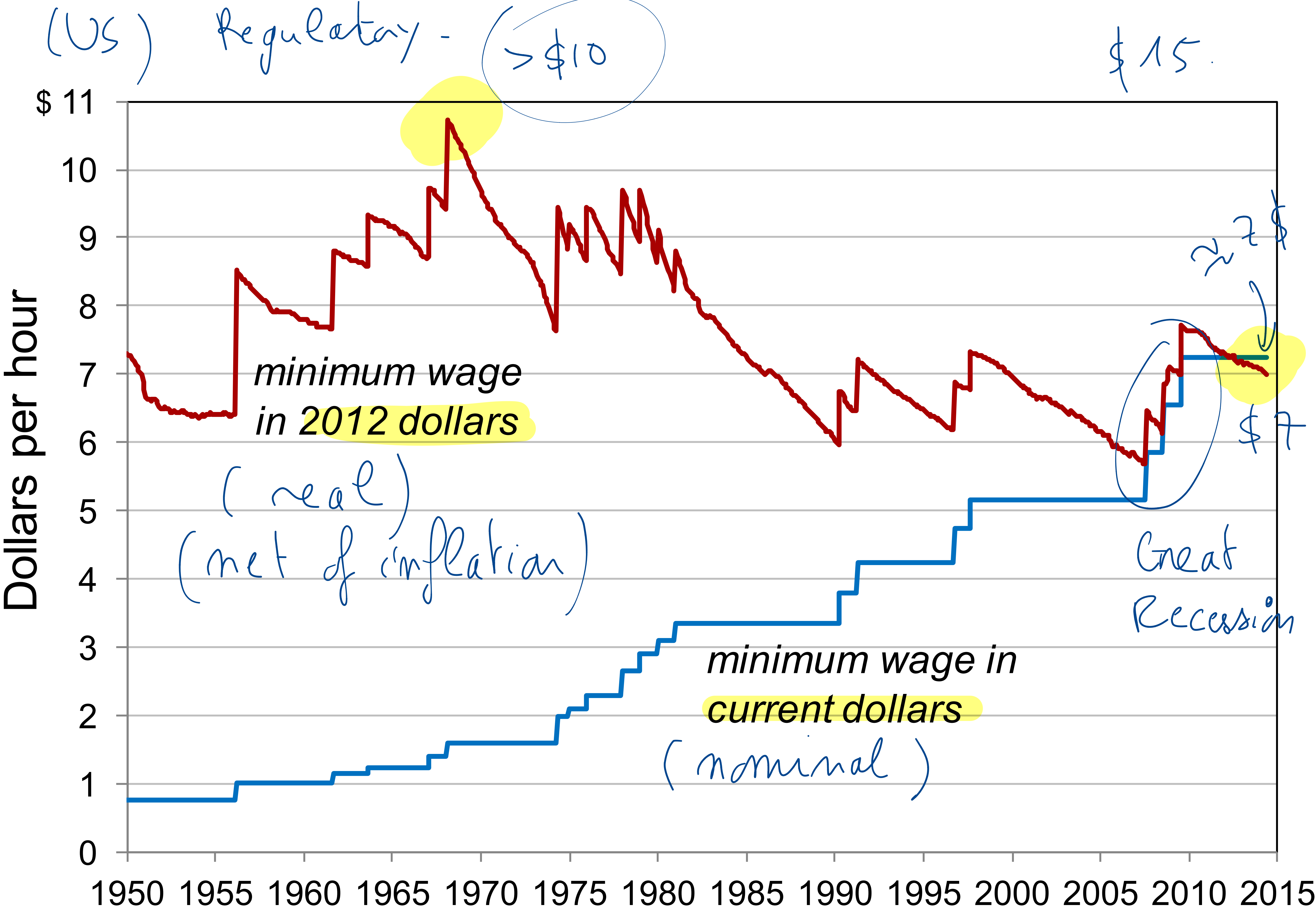
institutions

Union membership (US) selected years	
year	percent of labor force
1930	12.0
1945	35.0
1954	35.0
1970	27.0
1983	20.1
2013	11.3

(US)

industry	***# employed (1000s)***	***U % of total***	***wage ratio***
Private sector (total)	104,737	6.9	122.6
Government (total)	20,450	37.0	121.1
Construction	6,244	14.0	151.7
Mining	780	7.2	96.4
Manufacturing	13,599	10.5	107.2
Retail trade	14,582	4.9	102.4
Transportation	4,355	20.4	123.5
Finance, insurance	6,111	1.1	90.2
Professional services	12,171	2.1	99.1
Education	4,020	13.0	112.6
Health care	15,835	7.5	114.9

wage ratio = 100 × (union wage) / (nonunion wage)



(US) Regulatory - >$10
$15.
$ 11
10
9
8
7
6
5
4
3
2
1
0
Dollars per hour
minimum wage
in 2012 dollars
(real)
(net of inflation)
minimum wage in
current dollars
(nominal)
≈ 7 $
$7
Great
Recession
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015

Annual Turnover and Layoff Rates (%) at Ford, 1913–1915

Managerial -

	1913	1914	1915
Turnover rate	370	54	16
Layoff rate	62	7	0.1

- In 1914, Henry Ford announced that his company would pay a minimum of $5 a day for an eight-hour day, compared to an average of $2.30 for a nine-hour day previously.

- "There was no charity involved. We wanted to pay these wages so that the business would be on a lasting foundation. We were building for the future. A low wage business is always insecure. The payment of five dollars a day for an eight hour day was one of the finest cost cutting moves we ever made." Ford, *My Life and Work*, 1922.

Efficiency-wage theory : higher wages increase profits b/c they increase productivity more than costs

- workers are more dedicated to the firm (gift-exchange theory)
- working at the firm becomes more attractive compared to other firms

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Wage functions :

* Fixed wage : W is a parameter
  - does not change when other parameters change
  - does not change when θ changes
  - wage function in Hall (2005)

Advantages :
- simplicity
- wage is very rigid

→ wage does not absorb shocks, so U, V, θ will be very volatile, as we see in data.

<u>Disadvantage:</u> • in real world, wages respond somewhat to changes in labor productivity -> ω is not completely fixed.

× <u>Rigid wage:</u> wage function is

$$\boxed{W(a) = \omega \cdot a^{\gamma}}$$

a: labor productivity

ω: parameter capturing wage level

$\gamma \in [0,1]$: captures wage rigidity

$\gamma = 0$: $W = \omega$ -> <u>fixed wage</u>

$\gamma = 1$: $W = \omega \cdot a$ -> <u>flexible wage</u>

$0 < \gamma < 1$: wage is rigid

γ: <u>elasticity</u> of wage wrt labor productivity

$$\frac{d\ln W}{d\ln a} = \gamma$$ (percentage change in W when a changes by 1%)

γ in US data $\in [0.3, 0.7]$

$\gamma \approx 0.5$: reasonable estimate

- Blanchard & Gali (2010) $\gamma = 0.5$

- Michaillat (2012) $\gamma = 0.7$

× <u>Wage bargaining</u> (b/w worker & firm)

common bargaining solution: Nash bargaining (generalized)

here: <u>surplus-sharing solution</u>

P. Diamond (1982)

<u>Surplus sharing</u>:

- $\mathcal{F}$: surplus captured by firm
- $\mathcal{W}$: surplus captured by worker
- $\mathcal{T}$: total surplus from worker-firm match ($\mathcal{T} = \mathcal{F} + \mathcal{W}$)

$$\boxed{\begin{aligned} \mathcal{F} &= (1-\beta) \times \mathcal{T} \\ \mathcal{W} &= \beta \times \mathcal{T} \end{aligned}}$$

$\beta \in (0, 1)$: <u>bargaining power</u> of worker

- MPL: marginal product of labor

$$MPL = a \cdot \alpha \cdot N^{\alpha - 1} \quad (\alpha \in (0,1))$$

$$MPL = a \quad (\alpha = 1, \text{ linear production function})$$

- FOC from profit maximization:

$$MPL - (1 + \tau(\theta)) \cdot w = 0$$

$$\Rightarrow \quad MPL = (1+\tau) \cdot w$$

where τ : recruiter-producer ratio

$$\tau = r \cdot s / [q(\theta) - r \cdot s]$$

- z : value of unemployment (for workers)
  - unemployment benefits
  - leisure
  - home production

  $\Big\} \; z > 0$

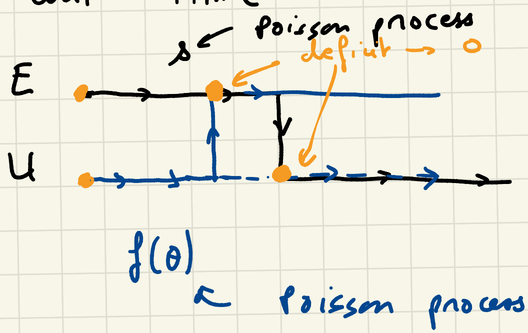
  - lower mental health / physical health from trauma of unemployment $\} \; z < 0$

- <u>what is firm surplus?</u> (in equilibrium)
  - output from the worker: MPL
  - cost of the worker: w

  ↳ firm earns $MPL - w$ per unit time.
  - Poisson process w/ arrival rate s destroys jobs → expected duration of worker-firm match is $1/s$.

- expected surplus from worker-firm match:

$$\boxed{\mathcal{F} = \frac{MPL - w}{s}}$$

- <u>what is worker's surplus?</u>
  - if worker is employed: w
  - if worker is unemployed: z (in units of output)
  - utility gain from employment: $w - z$ per unit time
  - 

as soon as employed loses a job, or unemployed finds a job: value from starting employed = 0.

[min(Poisson process λ_1, Poisson process λ_2)
→ Poisson process $\lambda_1 + \lambda_2$]

<u>Poisson process</u> with rate $\underline{s + f(\theta)}$ → employed & unemployed workers are in same situation.

→ $1/(s+f(\theta))$: expected duration of situation in which employed ≠ unemployed.

expected surplus from being employed:

$$\boxed{W = \frac{w - z}{s + f(\theta)}}$$

- Wage from surplus-sharing:

$$\left.\begin{aligned} F &= (1-\beta)\cdot S \\ W &= \beta \cdot S \end{aligned}\right\} \quad F = \frac{1-\beta}{\beta} \times W$$

$$\cancel{\beta} \cdot \frac{MPL - w}{s} \, (s + f(\theta)) = \frac{1-\beta}{\cancel{\beta}} \cdot \frac{w - z}{\cancel{s + f(\theta)}}$$

$$(1-\beta)(w - z) = \beta \cdot \left[1 + \frac{f(\theta)}{s}\right] \cdot (MPL - w)$$

$$(1-\beta) w - (1-\beta) z = \beta\, MPL - \beta w + \beta \frac{f(\theta)}{s} (MPL - w)$$

$$w = (1-\beta) z + \beta\, MPL + \beta \frac{f(\theta)}{s} (MPL - w)$$

$\tau(\theta) \cdot w$

$$w = (1-\beta) z + \beta\, MPL + \beta \cdot \boxed{\frac{f(\theta) \cdot \tau(\theta)}{s}} \cdot w$$

$$\tau(\theta) = \frac{r \cdot s}{q(\theta) - rs} \quad ; \quad f(\theta) = \theta \cdot q(\theta)$$

$$\frac{\tau(\theta) \cdot f(\theta)}{s} = \frac{r \cdot f(\theta)}{q(\theta) - rs} = r \cdot \theta \cdot \frac{q(\theta)}{q(\theta) - rs}$$

$$\frac{\tau(\theta) f(\theta)}{s} = r \cdot \theta \cdot \left[1 + \frac{rs}{q(\theta) - rs}\right]$$

$$\frac{\tau(\theta) f(\theta)}{s} = r \cdot \theta \cdot \left[1 + \tau(\theta)\right]$$

$$w = (1-\beta) z + \beta \, MPL + \beta r \theta \underbrace{[1 + \tau(\theta)] \, w}_{MPL}$$

$$\boxed{w = (1-\beta) z + \beta \cdot MPL \cdot (1 + r\theta)}$$

surplus-sharing solution to bargaining pb yields wage function

$$w(\underline{\beta}, \underline{z}, \underline{MPL}, \underline{\theta}, \underline{r})$$

- Pissarides (2000): Nash bargaining yields exactly same function as surplus sharing (eq. (1.20))
- if workers have all bargaining power:

$\beta = 1$ & $w = MPL(1 + r \cdot \theta)$

$w \geq MPL$ for any θ

→ no firms operate.

- if firms have all bargaining power: $\beta = 0$

$W = z$

- $0 < \beta < 1$:
  - $w \uparrow$ if $z \uparrow$ (better <u>outside option</u> for workers)
  - $w \uparrow$ if $MPL \uparrow$
  - $W \uparrow$ if $\theta \uparrow$