

# OPTIMAL PUBLIC EXPENDITURE WITH INEFFICIENT UNEMPLOYMENT

---

Pascal Michailat, Emmanuel Saez

Review of Economic Studies, 2019

Paper available at <https://pascalnichailat.org/6/>

# MAIN STABILIZATION POLICY: MONETARY POLICY

- policymakers rely primarily on monetary policy for stabilization
  - accordingly: extensive research on optimal monetary policy
- but monetary policy is sometimes constrained
  - zero lower bound (Japan, USA, EU)
  - monetary union (EU, USA)
  - ⇒ high unemployment
- then other stabilization policies are needed
  - but: very little is known about these alternative policies

# THIS PAPER: OPTIMAL PUBLIC EXPENDITURE

- public expenditure is commonly used for stabilization
  - US: Great Depression (New Deal), Great Recession (ARRA)
- framework: matching model from Michailat & Saez (2015)
- outcome: formula linking optimal stimulus spending to 3 sufficient statistics
  1. unemployment gap
  2. unemployment multiplier
  3. elasticity of substitution between public consumption & private consumption

## OPTIMAL PUBLIC EXPENDITURE: EXISTING RESULTS

- Samuelson (1954):
  - public goods financed by lump-sum taxation
  - efficient level of production
  - rule: spend until marginal utilities are equalized
  - but: what if production is inefficient?
- Keynes (1936):
  - no tradeoffs between public consumption & private consumption (multiplier  $> 1$ )
  - rule: spend to fill output gap
  - but: what if there is a tradeoff?
- our theory blends the theories of Samuelson & Keynes

# INFORMAL DESCRIPTION OF THE MODEL

---

# A SERVICE ECONOMY, WITHOUT FIRMS



# A SERVICE ECONOMY, WITHOUT FIRMS



# AN ASSET FOR SAVING





# PRIVATE SERVICES (c) & PUBLIC SERVICES (g)



# PRIVATE SERVICES (c) & PUBLIC SERVICES (g)



## MATCHING: NOT ALL SERVICES ARE SOLD



## MATCHING: NOT ALL SERVICES ARE SOLD



# MATCHING: COSTLY TO PURCHASE SERVICES

[Contact](#) [Site Map](#) [Home](#)

[Home](#) | [Services](#) | [Employers](#) | [Job Seekers](#) | [About](#) | [Contact](#)



Aunt Ann's  
IN-HOUSE STAFFING

## Staffing the Bay Area's finest homes

since 1958

San Francisco & Marin & Sonoma Counties: 415 749-3650  
East Bay Counties: 925 933-2273  
National Placements: 866 729-2667

Welcome to Aunt Ann's In House Staffing Nanny, Housekeeper and Estate Agency of San Francisco, CA - the nationwide leader in Household Staffing.

**EMPLOYERS**  
[Browse Candidates >>](#)

**JOB SEEKERS**  
[Browse Jobs >>](#)

Services

**Childcare Staffing**

- [Baby Nannies](#)
- [Family Assistants](#)
- [Governesses](#)
- [Nannies](#)
- [Parents Helpers](#)
- [On Call Nannies & Baby Sitters](#)

**Household Staffing**

- [Housekeepers](#)
- [Executive Housekeepers](#)
- [Houseman/Handyman](#)
- [Landscapers](#)
- [Cooks](#)
- [Cooks/Housekeepers](#)
- [Companions](#)

**Estate Staffing**

- [Estate Managers](#)
- [Household Managers](#)
- [Butlers](#)
- [Chefs](#)
- [Chauffeurs](#)

Aunt Ann's In House Staffing has been specializing in exclusive domestic staffing services for 50 years. We have worked with some of the world's finest families and estates, representing highly trained and qualified domestic household personnel.

Aunt Ann's In House Staffing is a full-service private service placement firm that specializes in the staffing of key private service positions including nannies, housekeepers, baby nurses, doulas, executive house keepers, private chefs, chauffeurs, butlers, household managers, domestic couples, estate managers, companions, personal assistants, executive assistants and more.



*Helping You through  
All the Stages  
of Your Life*



Aunt Ann's In House Staffing of San Francisco is staffed with seasoned, accomplished and professional placement consultants.

Our professional consultants are individuals who are trained to listen to your unique needs and are experienced in private service. We understand high standards and the art of service, absolute discretion and the special circumstances of our client's lives.



*Our History*  
Experience you can count on

Whether you are experienced in domestic service, or are a client searching to fill a position, we welcome the opportunity to offer our world class services to assist you.

# MATCHING: COSTLY TO PURCHASE SERVICES

## FEE SCHEDULE:

### *Fee for Long Term Services:*

#### CHILDCARE

- ♦ All **Full Time** Nanny, Parent Helper, Family Assistant, Governess  
15% of annual Gross Compensation (minimum fee = \$3000)
- ♦ All **Part Time** Nanny, Parent Helper, Family Assistant, Governess  
15% of annual Gross Compensation (minimum fee = \$1500)

#### HOUSEHOLD

- ♦ All **Full Time** Housekeeper, Executive Housekeeper, Cook, Handyman, Companion  
15% of annual Gross Compensation (minimum fee = \$3000)
- ♦ All **Part Time** Housekeeper, Executive Housekeeper, Cook, Handyman, Companion  
15% of annual Gross Compensation (minimum fee = \$1500)

#### ESTATE/ PRIVATE OFFICE

- ♦ All **Full Time and Part Time** Estate Managers, Household Managers, Chefs, Valets, Butlers, Master Gardeners, Security Body Guards, Chauffeurs, Couples, Personal Assistants, Executive Assistant Candidates  
20% of annual Gross Compensation (minimum fee = \$3000)

### *Fee for On-Call & Temporary Services*

- ♦ All On-Call and Temporary Work Assignments except for Baby Nurses, Newborn Specialists and Doulas  
35% of ongoing Gross Compensation (minimum fee = \$35 a day)
- ♦ All Baby Nurses, Newborn Specialists & Doulas  
20% of ongoing Gross Compensation (minimum fee = \$50 a day)

# SOCIALLY EFFICIENT RATE OF UNEMPLOYMENT

- too much unemployment is bad
    - too many services are idle
  - too little unemployment is bad
    - too many services are devoted to recruiting
  - there is a socially efficient rate of unemployment ( $u^*$ )
    - number of services enjoyed ( $y = g + c$ ) is maximized
- ⇒ when unemployment is efficient, Samuelson rule holds

# FORMAL DESCRIPTION OF THE MODEL

---



# STRUCTURE

- dynamic matching model
  - building on Michailat & Saez (2015)
- identical, self-employed households
- government
- 2 consumption goods traded on a matching market
  - public services & private services
- 1 asset for saving

# MATCHING MARKET

- capacity of each household:  $k$  services
- household purchases:  $C(t)$  private services
- government purchases:  $G(t)$  public services
- output:  $Y(t) = C(t) + G(t) < k$
- unemployment rate:  $u(t) = 1 - Y(t)/k$
- price of services:  $p(t)$

## MATCHING FUNCTION

- number of vacancies:  $v(t)$
- matching function:  $h(t) = \omega \cdot [k - Y(t)]^\eta \cdot v(t)^{1-\eta}$
- market tightness:  $x(t) = v(t)/(k - Y(t))$
- selling rate & buying rate:

$$f(x(t)) = \frac{h(t)}{k - Y(t)} = \omega \cdot x(t)^{1-\eta}$$

$$q(x(t)) = \frac{h(t)}{v(t)} = \omega \cdot x(t)^{-\eta}$$

# MARKET FLOWS

- relationships separate at rate  $s$
- given  $x$ , output and unemployment converge to

$$Y(x, k) = \frac{f(x)}{s + f(x)} \cdot k, \quad u(x) = \frac{s}{s + f(x)}$$

- convergence to steady state is extremely fast, so we assume:
  - $Y(t) = Y(x(t), k)$
  - $u(t) = u(x(t))$
  - see Hall (2005)

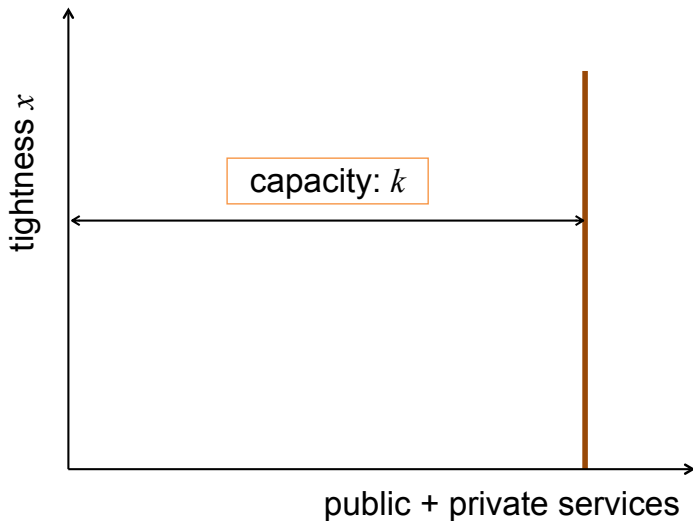
## MATCHING COST: $\rho$ SERVICES PER VACANCY

- output ( $Y$ ) = consumption ( $y$ ) + matching cost

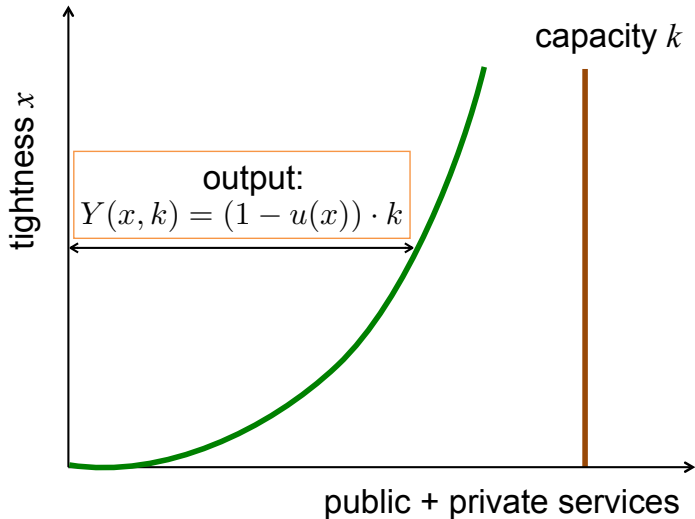
$$Y = y + \rho \cdot v = y + s \cdot Y \cdot \frac{\rho}{q(x)}$$

- matching wedge:  $\tau(x) = s \cdot \rho / [q(x) - s \cdot \rho]$
- total consumption:  $y = Y / [1 + \tau(x)]$
- private consumption:  $c = C / [1 + \tau(x)]$
- public consumption:  $g = G / [1 + \tau(x)]$

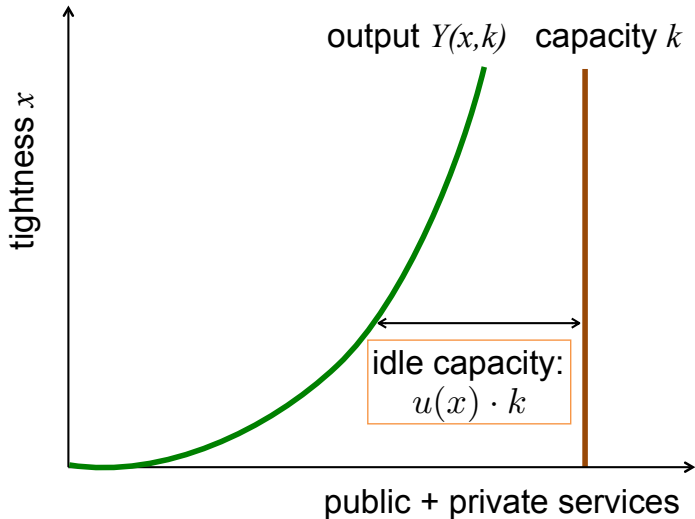
## SUPPLY STRUCTURE: SUMMARY



## SUPPLY STRUCTURE: SUMMARY

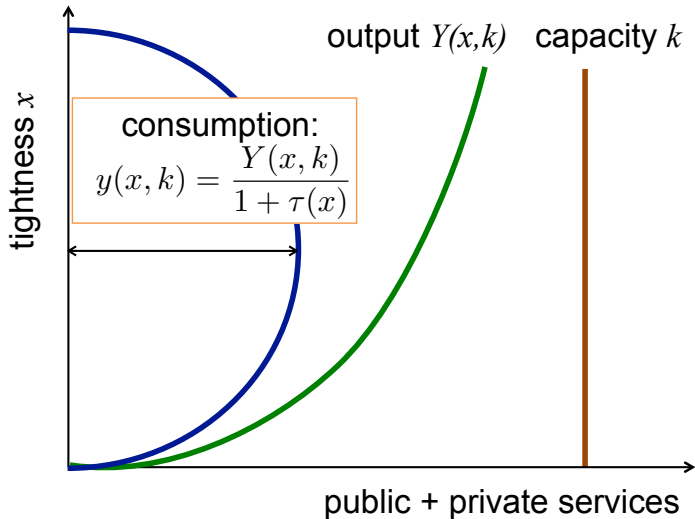


## SUPPLY STRUCTURE: SUMMARY

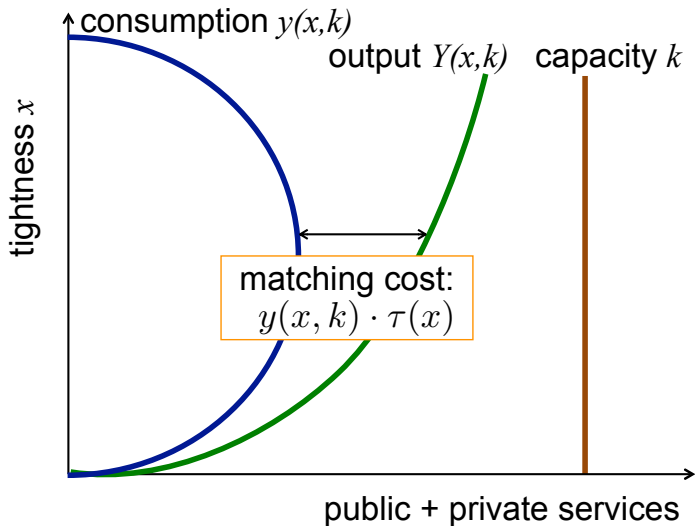




## SUPPLY STRUCTURE: SUMMARY



## SUPPLY STRUCTURE: SUMMARY



## DEMAND STRUCTURE: EXAMPLE

- asset: land  $l(t)$  in fixed supply  $l_0$ 
  - traded on a competitive market
  - Iacoviello (2005) and Liu, Wang, Zha (2013)
- households choose  $c(t)$  and  $l(t)$  to maximize utility

$$\int_0^{+\infty} e^{-\delta \cdot t} \cdot [\mathcal{U}(c, g) + \mathcal{V}(l)] dt$$

- subject to flow budget constraint

$$\dot{l} = p \cdot [1 - u(x)] \cdot k - p \cdot [1 + \tau(x)] \cdot c - T$$

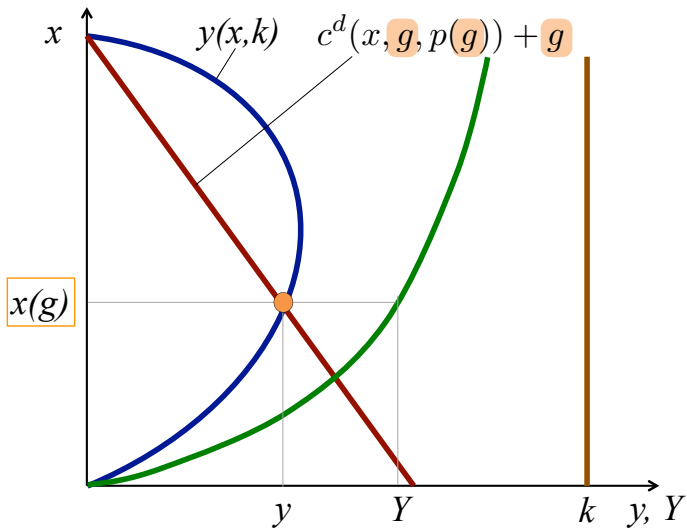
## AGGREGATE DEMAND IN THE EXAMPLE

- market clearing on housing market:  $l = l_0$
- private demand  $c^d(x, g, p)$  is solution to Euler equation:

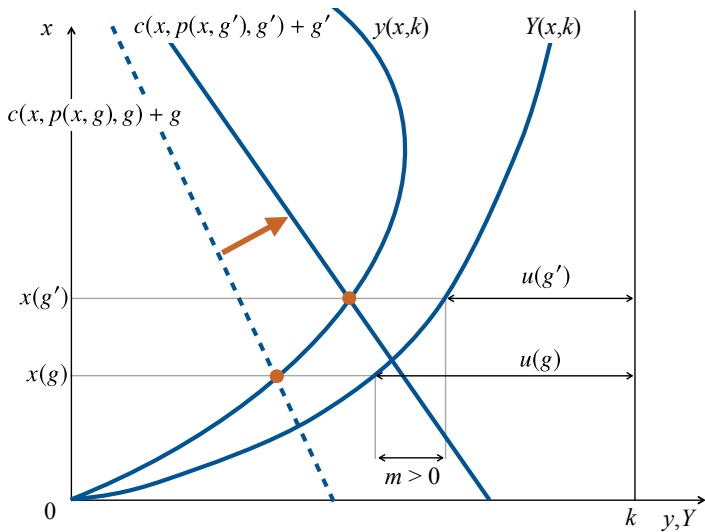
$$\frac{\partial \mathcal{U}}{\partial c}(c, g) = \frac{p \cdot (1 + \tau(x)) \cdot \mathcal{V}'(l_0)}{\delta}$$

- price of services relative to housing:  $p = p(x, g)$ 
  - general price mechanism
  - (assumption required in matching model)

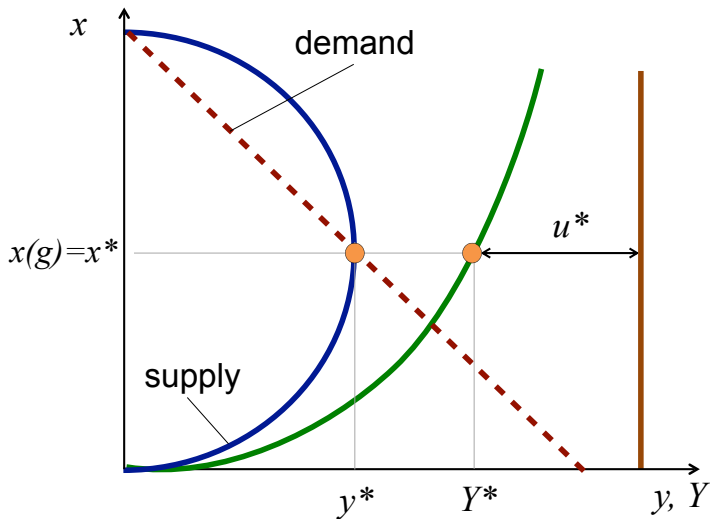
# EQUILIBRIUM TIGHTNESS $x(g)$



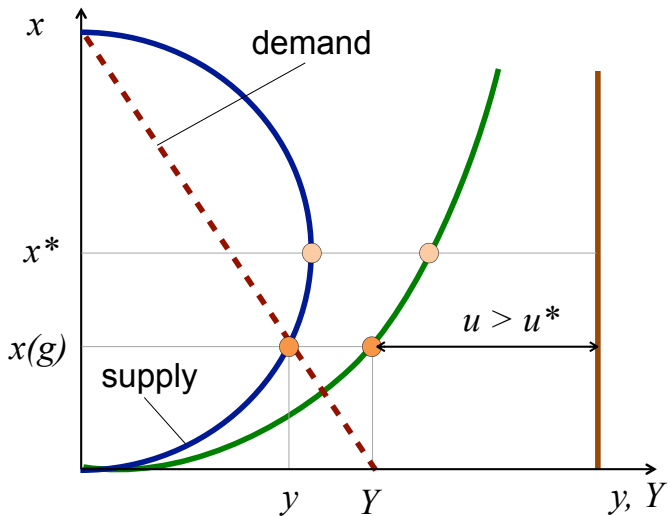
# UNEMPLOYMENT MULTIPLIER $m$



# SOCIALLY EFFICIENT UNEMPLOYMENT RATE $u^*$

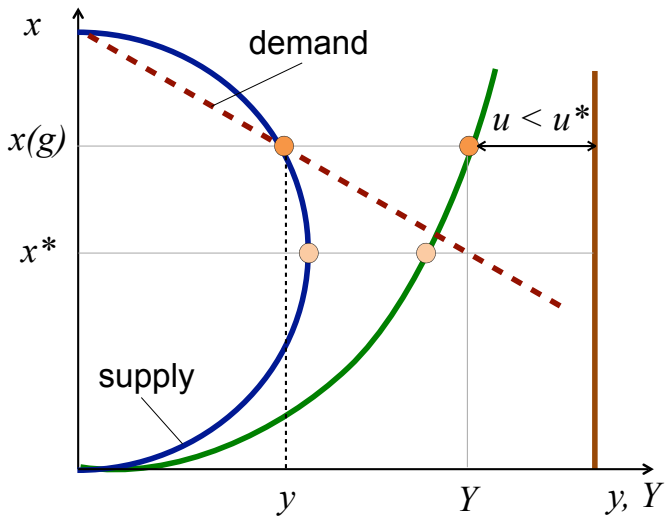


# INEFFICIENTLY HIGH UNEMPLOYMENT RATE





# INEFFICIENTLY LOW UNEMPLOYMENT RATE



# OPTIMAL PUBLIC EXPENDITURE

---

# GOVERNMENT'S PROBLEM

- households' flow utility is  $\mathcal{U}(c, g)$
- public expenditure is financed by a lump-sum tax to maintain a balanced budget
- given  $x(g)$ , the government chooses  $g$  to maximize

$$\mathcal{U}\left(\underbrace{y(x(g), k) - g}_c, g\right)$$

## CORRECTING THE SAMUELSON FORMULA

- first-order condition of government's problem is

$$0 = \frac{\partial \mathcal{U}}{\partial g} - \frac{\partial \mathcal{U}}{\partial c} + \frac{\partial \mathcal{U}}{\partial c} \cdot \frac{\partial y}{\partial x} \cdot \frac{dx}{dg}$$

- optimal public expenditure satisfies

$$\underbrace{1 = MRS_{gc}}_{\text{Samuelson formula}} + \underbrace{\frac{\partial y}{\partial x} \cdot \frac{dx}{dg}}_{\text{correction}}$$

- $MRS_{gc} = (\partial \mathcal{U} / \partial g) / (\partial \mathcal{U} / \partial c)$
- correction due to effect of public expenditure on welfare through tightness

## INTRODUCING ESTIMABLE STATISTICS

- $(g/c)^*$ : Samuelson spending
- elasticity of substitution between  $g$  and  $c$ :

$$1 - MRS_{gc} \approx \frac{1}{\epsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$

- unemployment gap:

$$\frac{\partial y}{\partial x} \propto u - u^*$$

- unemployment multiplier:

$$\frac{dx}{dg} \propto m = -\frac{y}{1-u} \cdot \frac{du}{dg}$$

## IMPLICIT FORMULA FOR OPTIMAL STIMULUS

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx z_0 \epsilon m \cdot \frac{u - u^*}{u^*}$$

- $g/c - (g/c)^*$ : stimulus spending
- $\epsilon$ : elasticity of substitution between  $g$  and  $c$ 
  - = marginal social value of public spending
- $m$ : unemployment multiplier
  - decrease in  $u$  when  $g$  increases by 1% of  $y$
- $u - u^*$ : unemployment gap
  - = productive inefficiency
- $z_0$ : constant of the parameters  $\eta, u^*$

## DEPARTURES FROM SAMUELSON RULE

	$m < 0$	$m = 0$	$m > 0$
$u > u^*$	$g/c < (g/c)^*$	$g/c = (g/c)^*$	$g/c > (g/c)^*$
$u = u^*$	$g/c = (g/c)^*$	$g/c = (g/c)^*$	$g/c = (g/c)^*$
$u < u^*$	$g/c > (g/c)^*$	$g/c = (g/c)^*$	$g/c < (g/c)^*$

# MARGINAL VALUE OF PUBLIC SERVICES

- $\epsilon = 0$ : digging holes or building pyramids
  - $g/c = (g/c)^*$ : Samuelson rule holds, no stimulus spending
- $\epsilon \rightarrow +\infty$ : perfect substitution
  - $u = u^*$ : entirely fill unemployment gap, as in Keynes
- $\epsilon \in (0, +\infty)$ : medium substitution
  - medium stabilization:  $g/c \neq (g/c)^*$  but  $u \neq u^*$   
~> partially fill unemployment gap



## MAKING THE FORMULA EXPLICIT

- implicit formula: not useful for quantitative results because  $u$  in RHS responds to  $g/c$  in LHS
- starting from  $(g/c)^*$  and  $u_0 \neq u^*$ :

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx z_0 \epsilon m \cdot \frac{u(g/c) - u^*}{u^*}$$

- first-order Taylor expansion of  $u$  at  $u((g/c)^*) = u_0$ :

$$\frac{u - u^*}{u^*} \approx \frac{u_0 - u^*}{u^*} - z_1 m \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$

- $z_1$ : constant of the parameters  $u^*$ ,  $(g/c)^*$

## EXPLICIT FORMULA

- optimal  $g/c$  depends on fixed quantities:

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{z_0 \epsilon m}{1 + z_1 z_0 \epsilon m^2} \cdot \frac{u_0 - u^*}{u^*}$$

- optimal  $u$  depends on fixed quantities:

$$u \approx u^* + \frac{u_0 - u^*}{1 + z_1 z_0 \epsilon m^2}$$

- approximations valid up to 2nd-order terms

## RESULTS WITH DISTORTIONARY TAXATION

- endogenous capacity:  $\mathcal{U}(c, g, k)$  with  $\partial \mathcal{U} / \partial k < 0$
- linear income tax:  $T = \tau^L \cdot (1 - u(x)) \cdot k$
- everything remains valid
  - but  $(g/c)^*$  is lower because of tax distortions
- however: link between multipliers changes
  - no tax distortions:  $m = dY/dG$
  - tax distortions:  $m > dY/dG$
  - with taxes, we may have  $dY/dG < 0$  but  $m > 0$

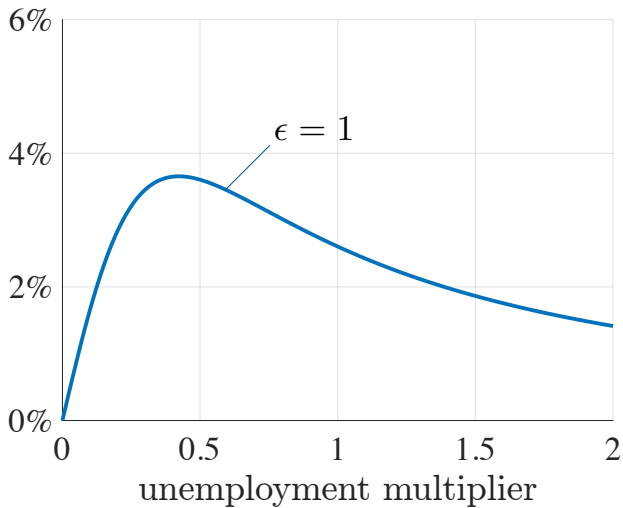
NUMERICAL ILLUSTRATION:  
GREAT RECESSION IN THE US

---

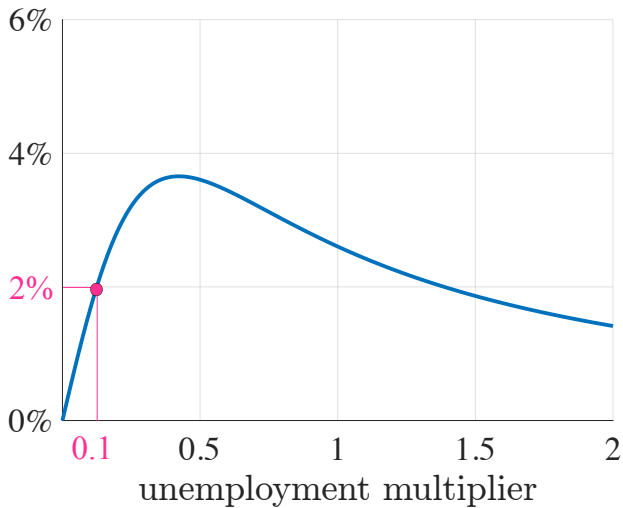
## STARTING POINT: WINTER 2008–2009

- unemployment = 6% and public spending = 16.5% of GDP
  - for illustration: we take these values as efficient
- unemployment is forecast to increase to 9%
  - initial unemployment gap =  $9\% - 6\% = 3\%$
- we compute optimal stimulus for various elasticities of substitution and unemployment multipliers

# OPTIMAL STIMULUS SPENDING (% OF GDP)



## OPTIMAL STIMULUS SPENDING (% OF GDP)

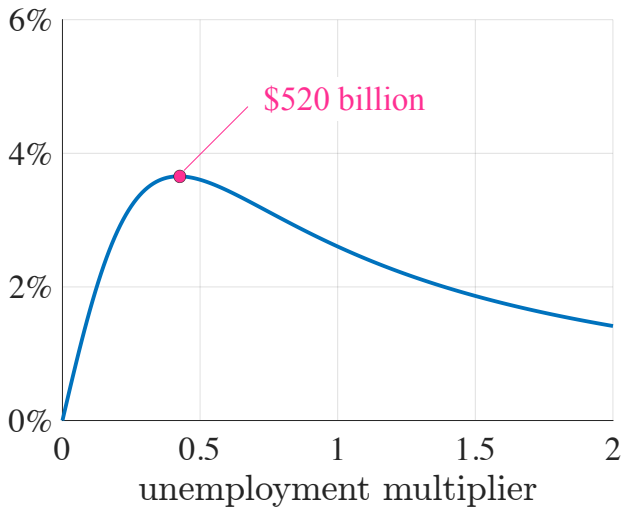


# OPTIMAL STIMULUS SPENDING (% OF GDP)

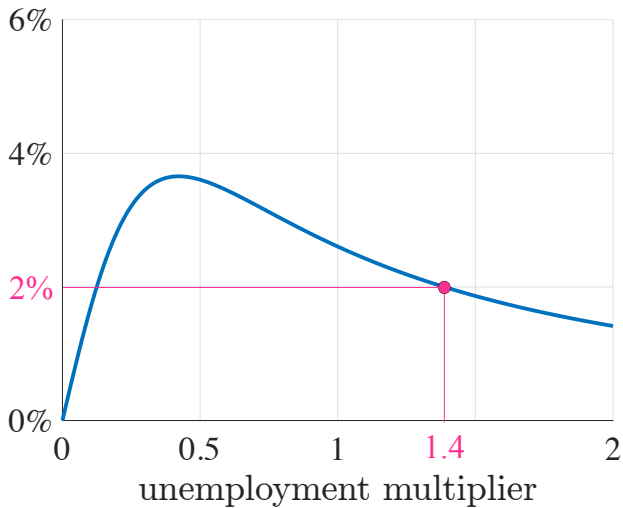




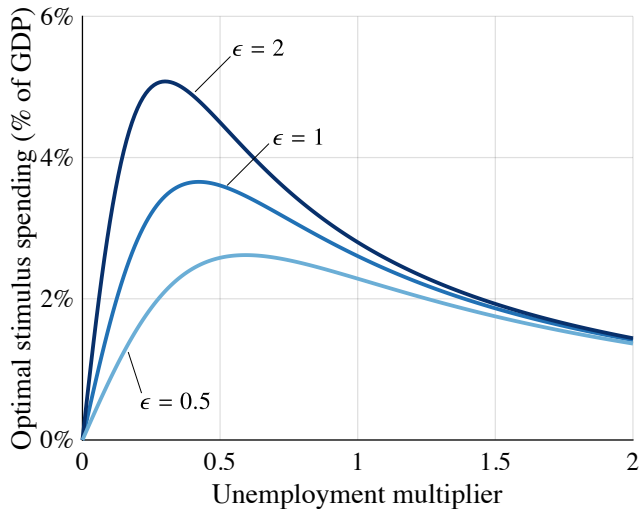
# OPTIMAL STIMULUS SPENDING (% OF GDP)



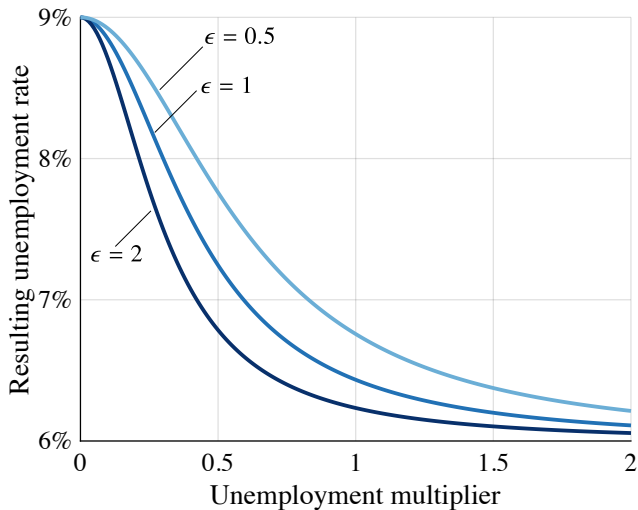
# OPTIMAL STIMULUS SPENDING (% OF GDP)



# OPTIMAL STIMULUS SPENDING FOR VARIOUS $\epsilon$



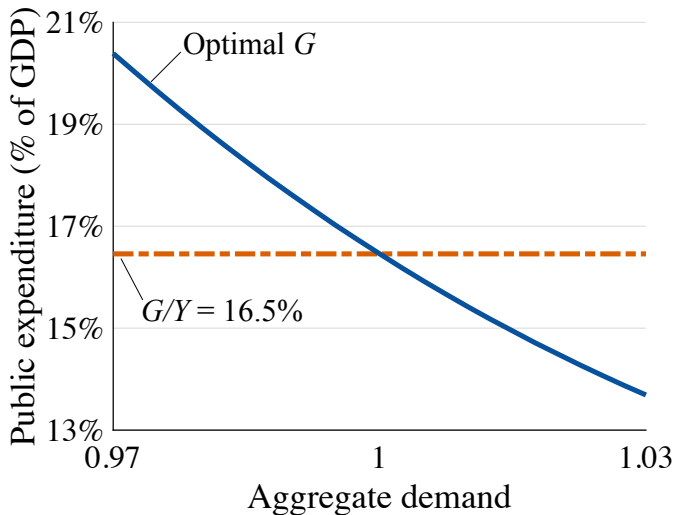
# UNEMPLOYMENT UNDER OPTIMAL STIMULUS



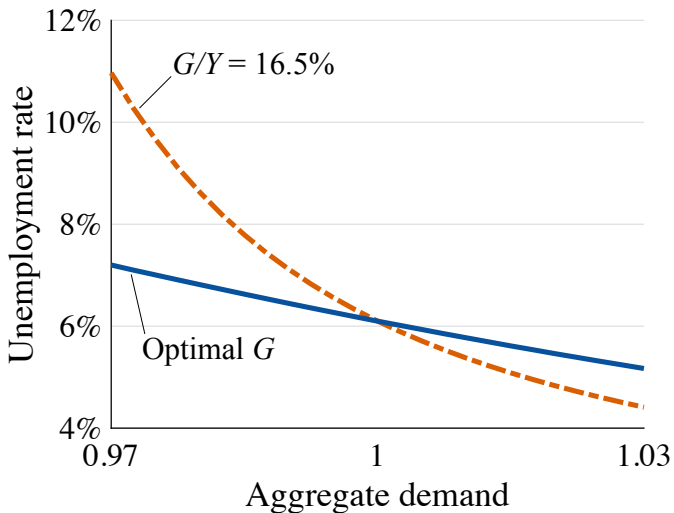
# SOME SIMULATIONS

---

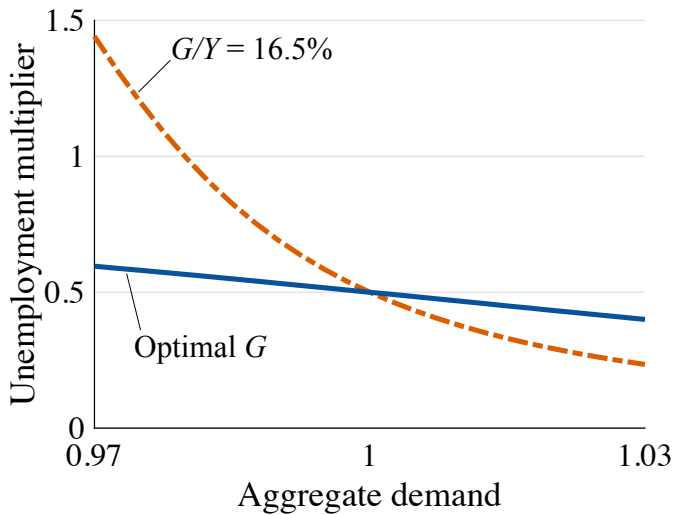
## OPTIMAL STIMULUS IN CALIBRATED MODEL



# UNEMPLOYMENT RATE IN CALIBRATED MODEL

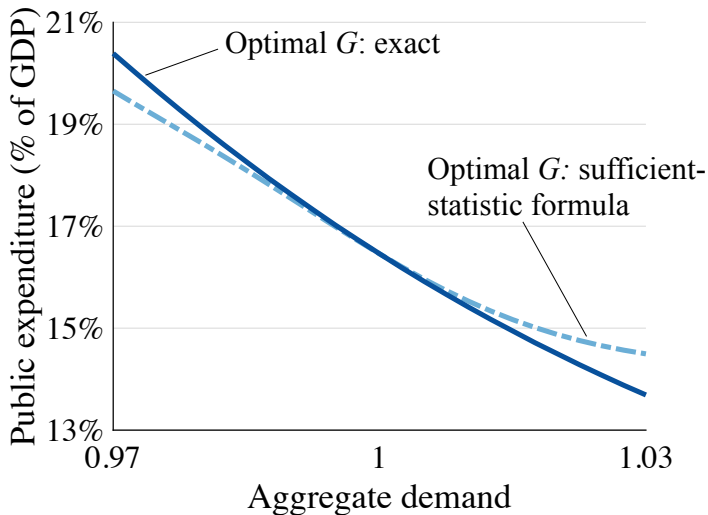


## MULTIPLIER IN CALIBRATED MODEL





## QUALITY OF APPROXIMATIONS IN FORMULA



# SUMMARY & DISCUSSION

---

1.  $dY/dG > 1$  is not necessary for stimulus
  - stimulus requires unemployment multiplier  $> 0$  (as in data)
2. bang-for-the-buck logic does not hold
  - strongest stimulus for  $m = 0.4$
  - same stimulus for  $m = 0.1$  and  $m = 1.4$
3. completely filling the unemployment gap is not optimal
  - optimal to partially fill unemployment gap
  - except if public services = private services
4. low marginal social value of  $g$  does not imply no stimulus
  - optimal to reduce unemployment gap
  - except if public services = digging holes

## DISTORTIONARY TAXES $\nRightarrow$ SMALLER STIMULUS

- formula remains valid with distortionary taxation
  - but Samuelson spending is lower
- however,  $dY/dG$  is not useful anymore because  $dY/dG \neq m$ 
  - $dY/dG = m +$  labor-supply response to taxes
  - labor-supply distortion reduces  $dY/dG$  but not  $m$
  - so:  $m > dY/dG$
  - possibly:  $dY/dG < 0$  while  $m > 0$
- distortionary taxation does not imply smaller stimulus
  - only average public spending is lower