AGGREGATE DEMAND, IDLE TIME, AND UNEMPLOYMENT: ONLINE APPENDICES

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ONLINE APPENDIX A: LONG PROOFS

Proof of Proposition 9. In a fixprice equilibrium parameterized by $p_0 > 0$ and $w_0 > 0$, (x, θ) satisfies $n^s(\theta) = n^d(\theta, x, w_0)$ and $c^s(x, \theta) = c^d(x, p_0)$. We look for equilibria with positive consumption. These equilibria necessarily have $\theta \in (0, \theta^m)$ and $x \in (0, x^m)$.

Equation $n^d(\theta, x, w_0) = n^s(\theta)$ is equivalent to $\left[n^d(\theta, x, w_0)\right]^{1-\alpha} = [n^s(\theta)]^{1-\alpha}$. Following the logic of the proof of Proposition 3, we can show that the latter equation is equivalent to

$$F(\theta, x, a, h, w) \equiv f(x) - \hat{f}(\theta)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta))^{\alpha} \cdot h^{1-\alpha} \cdot \frac{w}{\alpha \cdot a} = 0. \tag{A1}$$

Since $\alpha < 1$, the function $\theta \mapsto \hat{f}(\theta)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta))^{\alpha}$ is strictly increasing from 0 to $+\infty$ on $[0, \theta^m)$. Hence, (A1) implicitly defines θ as a function Θ^F of $x \in [0, +\infty)$. Since f is strictly increasing from 0 to 1 on $[0, +\infty)$, Θ^F is strictly increasing on $[0, +\infty)$, $\Theta^F(0) = 0$, and $\lim_{x \to +\infty} \Theta^F(x) = \theta^F$ where $\theta^F \in (0, \theta^m)$ is defined by $\hat{f}(\theta^F)^{1-\alpha} \cdot \left(1 + \hat{\tau}(\theta^F)\right)^{\alpha} \cdot h^{1-\alpha} \cdot w/(\alpha \cdot a) = 1$.

The proof of Proposition 3 shows that the equation $c^s(x, \theta) = c^d(x, p_0)$, combined with $k = a \cdot n^{\alpha}$ is equivalent to

$$f(x) \cdot a \cdot \left(\frac{\hat{f}(\theta)}{1 + \hat{\tau}(\theta)} \cdot h\right)^{\alpha} \cdot (1 + \tau(x))^{\varepsilon - 1} = \chi^{\varepsilon} \cdot \frac{\mu}{p}.$$

Using (A1), we transform this equation to

$$G(\theta, x, h, \chi, \mu, w, p) \equiv \hat{f}(\theta) \cdot (1 + \tau(x))^{\varepsilon - 1} - \frac{\alpha}{w \cdot h} \cdot \chi^{\varepsilon} \cdot \frac{\mu}{p} = 0. \tag{A2}$$

If $\alpha \cdot \chi^\varepsilon \cdot (\mu/p)/(w \cdot h) \geq 1$, we define $x^G(p,w)$ by $\left(1+\tau(x^G)\right)^{\varepsilon-1} = \alpha \cdot \chi^\varepsilon \cdot (\mu/p)/(w \cdot h)$. If $\alpha \cdot \chi^\varepsilon \cdot (\mu/p)/(w \cdot h) < 1$, we set $x^G(p,w) = 0$. Since $\varepsilon > 1$, the function $x \mapsto (1+\tau(x))^{\varepsilon-1}$ is strictly increasing from 1 to $+\infty$ on $[0,x^m)$; therefore, x^G is well defined and $x^G(p,w) \in (0,x^m)$. Since $\hat{f}$ is strictly increasing from 0 to 1 on $(0,+\infty)$, (A2) implicitly defines θ as a function Θ^G of $x \in (x^G(p,w),x^m)$. Moreover, Θ^G is strictly decreasing on $(x^G(p,w),x^m)$, $\lim_{x\to x^G(p,w)} \Theta^G(x) = +\infty$, and $\lim_{x\to x^m} \Theta^G(x) = 0$.

The system of (A1) and (A2) is equivalent to the system of $\Theta^F(x) = \Theta^G(x)$ and $\theta = \Theta^F(x)$. The properties of Θ^F and Θ^G imply that this system admits a unique solution (x,θ) with $x \in (x^G(p,w),x^m)$ and $\theta \in (0,\theta^F)$.

Proof of Proposition 10.

Aggregate Demand Shocks. We parameterize an increase in aggregate demand by an increase in χ or μ. The function F in (A1) satisfies $\partial F/\partial\theta < 0$, $\partial F/\partial x > 0$, $\partial F/\partial a > 0$, and $\partial F/\partial h < 0$. Using the implicit function theorem, we write the solution θ to $F(\theta,x,a,h,w) = 0$ as a function $\Theta^F(x,a,h)$ with $\partial\Theta^F/\partial x > 0$, $\partial\Theta^F/\partial a > 0$, and $\partial\Theta^F/\partial h < 0$.

The function G in (A2) satisfies $\partial G/\partial\theta > 0$, $\partial G/\partial x > 0$, $\partial G/\partial h > 0$, $\partial G/\partial\chi < 0$, and $\partial G/\partial\mu < 0$. Using the implicit function theorem, we write the solution θ to $G(\theta,x,h,\chi,\mu,w,p) = 0$ as a function $\Theta^G(x,h,\chi,\mu)$ with $\partial\Theta^G/\partial x < 0$, $\partial\Theta^G/\partial h < 0$, $\partial\Theta^G/\partial\chi > 0$, and $\partial\Theta^G/\partial\mu > 0$.

In equilibrium, x satisfies $G(\Theta^F(x,a,h),x,h,\chi,\mu) = 0$. Given that $\partial\Theta^F/\partial x > 0$, $\partial G/\partial\theta > 0$, $\partial G/\partial x > 0$, and $\partial G/\partial\chi < 0$, the implicit function theorem implies that $\partial x/\partial\chi > 0$. We can show similarly that $\partial x/\partial\mu > 0$. Since $\theta = \Theta^F(x,a,h)$ with $\partial\Theta^F/\partial x > 0$, we also have $\partial\theta/\partial\chi > 0$ and $\partial\theta/\partial\mu > 0$. Equation (8) yields $y = \hat{f}(\theta) \cdot h \cdot w/\alpha$; therefore, the comparative statics for θ imply that $\partial y/\partial\chi > 0$ and $\partial y/\partial\mu > 0$. Since $l = \hat{f}(\theta) \cdot h$, the comparative statics for θ also imply that $\partial l/\partial\chi > 0$ and $\partial l/\partial\mu > 0$.

Technology Shocks. We parameterize an increase in technology by an increase in a. In equilibrium, x satisfies $F(\Theta^G(x,h,\chi,\mu),x,a,h) = 0$. Given that $\partial\Theta^G/\partial x < 0$, $\partial F/\partial\theta < 0$, $\partial F/\partial x > 0$, and $\partial F/\partial a > 0$, the implicit function theorem implies that $\partial x/\partial a < 0$. Since $\theta = \Theta^G(x,h,\chi,\mu)$ with $\partial\Theta^G/\partial x < 0$, we obtain $\partial\theta/\partial a > 0$. The logic presented for aggregate demand shocks implies that since $\partial\theta/\partial a > 0$, then $\partial y/\partial a > 0$ and $\partial l/\partial a > 0$.

Labor Supply Shocks. We parameterize an increase in labor supply by an increase in h. The functions F and G both depend on h, so it is impossible to obtain comparative statics for x and θ from them. To obtain the comparative statics, we manipulate and combine (A1) and (A2), and we obtain

$$H(\theta,x) \equiv (1+\hat{\tau}(\theta))^{\alpha} - f(x)\cdot(1+\tau(x))^{(1-\alpha)\cdot(\varepsilon-1)}\cdot a\cdot\left(\frac{\alpha}{w}\right)^{\alpha}\cdot\left(\chi^{\varepsilon}\cdot\frac{\mu}{p}\right)^{\alpha-1} = 0. \tag{A3}$$

The function H satisfies $\partial H/\partial\theta > 0$ and $\partial H/\partial x < 0$. The function H does not depend on h, which resolves the earlier problem. Using the implicit function theorem, we write the solution θ to $H(\theta,x) = 0$ as a function $\Theta^H(x)$ with $\partial\Theta^H/\partial x > 0$.

In equilibrium, x satisfies $G(\Theta^H(x),x,h) = 0$. Given that $\partial\Theta^H/\partial x > 0$, $\partial G/\partial\theta > 0$, $\partial G/\partial x > 0$, and $\partial G/\partial h > 0$, the implicit function theorem implies that $\partial x/\partial h < 0$. Since $\theta = \Theta^H(x)$ with $\partial\Theta^H/\partial x > 0$, we obtain $\partial\theta/\partial h < 0$. We find that $\partial y/\partial h > 0$ because $y = (1+\tau(x))\cdot c^d(x,p) = (1+\tau(x))^{1-\varepsilon}\cdot\chi^{\varepsilon}\cdot\mu/p$ and $1-\varepsilon < 0$ and $\partial x/\partial h < 0$. We also find that $\partial l/\partial h > 0$ because $l = \alpha\cdot y/w$ (from equation (8)) and $\partial y/\partial h > 0$.

Mismatch Shocks. We parameterize an increase in mismatch by a decrease in matching efficacy on the labor market along with a corresponding decrease in recruiting cost: $\hat{f}(\theta)$, $\hat{q}(\theta)$, and ρ become $\lambda\cdot\hat{f}(\theta)$, $\lambda\cdot\hat{q}(\theta)$, and $\lambda\cdot\hat{\rho}$ with $\lambda < 1$. Consequently, the function $\hat{\tau}$ remains the same. With the parameter λ for mismatch, the functions H and Θ^H are the same, but the function G depends on λ with $\partial G/\partial\lambda > 0$. In equilibrium, x satisfies $G(\Theta^H(x),x,\lambda) = 0$. Given that $\partial\Theta^H/\partial x > 0$, $\partial G/\partial\theta > 0$, $\partial G/\partial x > 0$, and $\partial G/\partial\lambda > 0$, the implicit function theorem implies that $\partial x/\partial\lambda < 0$. Since $\theta = \Theta^H(x)$ with $\partial\Theta^H/\partial x > 0$, we have $\partial\theta/\partial\lambda < 0$. The logic presented for labor supply shocks implies that since $\partial x/\partial\lambda < 0$, then $\partial y/\partial\lambda > 0$ and $\partial l/\partial\lambda > 0$.

Comparative Statics with Partially Rigid Price and Real Wage. Using the expressions of the partially rigid price and real wage, we rewrite (A1), (A2), and (A3) as

$$f(x) - \hat{f}(\theta)^{1-\alpha} \cdot (1+\hat{\tau}(\theta))^{\alpha} \cdot \frac{w_0 \cdot h^{(1-\alpha)\cdot(1-\xi)}}{(\alpha \cdot a)^{1-\xi}} = 0$$

$$\hat{f}(\theta) \cdot (1+\tau(x))^{\varepsilon-1} - \frac{\alpha^{1-\xi}}{w_0 \cdot h^{1-\xi}} \cdot \frac{(\chi^{\varepsilon} \cdot \mu)^{1-\xi}}{p_0} = 0$$

$$(1+\hat{\tau}(\theta))^{\alpha} - f(x) \cdot (1+\tau(x))^{(1-\alpha)\cdot(\varepsilon-1)} \cdot a^{1-\xi} \cdot \frac{\alpha^{\alpha(1-\xi)}}{w_0^{\alpha}} \cdot \left[\frac{(\chi^{\varepsilon} \cdot \mu)^{1-\xi}}{p_0}\right]^{\alpha-1} = 0.$$

The implicit functions defined by these equations have exactly the same properties as the functions F, G, and H. Hence, the comparative statics for x and θ are the same in the fixprice equilibrium and in the equilibrium with partially rigid prices. Finally, the arguments used above for the fixprice equilibrium imply that the comparative statics for y and l are the same in the fixprice equilibrium and in the equilibrium with partially rigid prices.

Proof of Proposition 11. In a competitive equilibrium, the pair (p^*, w^*) satisfies $n^s(\theta^*) = n^d(\theta^*, x^*, w^*)$ and $c^s(x^*, \theta^*) = c^d(x^*, p^*)$. Following the steps of the proof of Proposition 9, we show that (p^*, w^*) satisfies

$$w^* = f(x^*) \cdot \hat{f}(\theta^*)^{\alpha-1} \cdot (1+\hat{\tau}(\theta^*))^{-\alpha} \cdot h^{\alpha-1} \cdot \alpha \cdot a$$

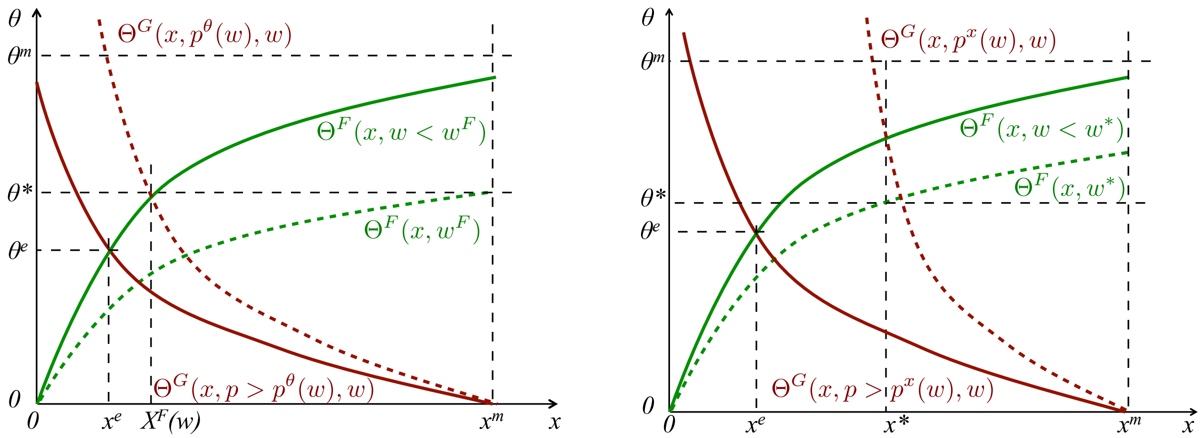
$$p^* = \frac{(1+\tau(x^*))^{1-\varepsilon}}{\hat{f}(\theta^*)} \cdot \frac{\alpha}{w^* \cdot h} \cdot \chi^{\varepsilon} \cdot \mu.$$

This system admits a unique solution. Thus, there exists a unique competitive equilibrium. Clearly, the wage w^* satisfies the expression in the proposition. Combining these two equations, we find that the price p^* also satisfies the expression in the proposition.

Online Appendix B: The Four Inefficient Regimes

This appendix establishes the boundaries in a (w, p) plane of the four inefficient regimes of the model of Section III. These boundaries are depicted in Figure VII.

Proposition A1. *There exists a function $w \mapsto p^x(w)$ such that for any $w > 0$, the product market is slack if $p > p^x(w)$ and tight if $p < p^x(w)$. There exists a function $w \mapsto p^{\theta}(w)$ such that for*



A. Condition such that $\theta < \theta^*$

B. Condition such that $x < x^*$

Figure A1: Illustration of the Proof of Proposition A1

any $w > 0$, the labor market is slack if $p > p^\theta(w)$ and tight if $p < p^\theta(w)$. The function p^x is strictly decreasing for $w \in (0, w^*]$, strictly increasing for $w \in [w^*, +\infty)$, $\lim_{w \to 0} p^x(w) = +\infty$, and $\lim_{w \to +\infty} p^x(w) = +\infty$. The function p^θ is strictly decreasing for $w \in (0, w^F]$, $\lim_{w \to 0} p^\theta(w) = +\infty$, and $p^\theta(w) = 0$ for $w \in [w^F, +\infty)$, where $w^F \in (w^*, +\infty)$. Furthermore, $p^x(w^*) = p^\theta(w^*) = p^*$.

Proof. We build on the proof of Proposition 9. We define the function $\Theta^F : [0, +\infty) \times (0, +\infty) \to (0, \theta^m)$ such that $F(\Theta^F(x, w), x, a, h, w) = 0$. The function Θ^F is strictly increasing in x and strictly decreasing in w. We define the function Θ^G: $\left\{(x, w, p) | p > 0,\ w > 0,\ x^G(p, w) < x < x^m\right\} \to (0, +\infty)$ such that $G(\Theta^G(x, w, p), x, h, \chi, \mu, w, p) = 0$. The function Θ^G is strictly decreasing in x, p, and w. The proof is illustrated in Figure A1.

First Part: Condition such that $\theta < \theta^*$. Let w^F be defined by $\Theta^F(x^m, w^F) = \theta^*$. For all $w > w^F$ and for all $x \in [0, x^m]$, $\Theta^F(x, w) < \theta^*$. For all $w \leq w^F$, there exists a unique $x \in [0, x^m]$ such that $\Theta^F(x, w) = \theta^*$. We define the function $X^F : (0, w^F] \to [0, x^m]$ by $\Theta^F(X^F(w), w) = \theta^*$. The function X^F is strictly increasing, $\lim_{w \to 0} X^F(w) = 0$, $X^F(w^*) = x^*$, and $X^F(w^F) = x^m$.

Next, for all $w \leq w^F$, there exists a unique $p \in (0, +\infty)$ such that $\Theta^G(X^F(w), p, w) = \theta^*$. We define the function $p^\theta : (0, w^F] \to (0, +\infty)$ by $\Theta^G(X^F(w), p^\theta(w), w) = \theta^*$. The function p^θ is strictly decreasing, $\lim_{w \to 0} p^\theta(w) = +\infty$, $p^\theta(w^*) = p^*$, and $p^\theta(w^F) = 0$. We extend the definition of p^θ by setting $p^\theta(w) = 0$ for all $w \in (w^F, +\infty)$.

Last, we denote equilibrium labor market tightness by θ^e and equilibrium product market tightness by x^e. For any $w > w^F$, $\theta^e = \Theta^F(x, w) < \theta^*$ by definition of w^F. Consider $w \leq w^F$ and

$p > p^{\theta}(w)$. Then $\Theta^G(X^F(w), p, w) < \Theta^G(X^F(w), p^{\theta}(w), w) = \theta^* = \Theta^F(X^F(w), w)$ because Θ^G is strictly decreasing in p. Given that Θ^F is strictly increasing in x and Θ^G is strictly decreasing in x and $\Theta^G(x^e, p, w) = \Theta^F(x^e, w)$, we conclude that $x^e < X^F(w)$. Thus, $\theta^e = \Theta^F(x^e, w) < \Theta^F(X^F(w), w) = \theta^*$ because Θ^F is strictly increasing in x. In sum, for any $w > 0$ and $p > p^{\theta}(w)$, we have $\theta^e < \theta^*$. Following the same logic, we find that for any $w > 0$ and $p < p^{\theta}(w)$, we have $\theta^e > \theta^*$.

Second Part: Condition such that $x < x^*$. We define the function $p^x : (0, +\infty) \to (0, +\infty)$ by

$$p^x(w) = \frac{(1 + \tau(x^*))^{1-\varepsilon}}{h \cdot \hat{f}(\Theta^F(x^*, w))} \cdot \chi^{\varepsilon} \cdot \frac{\alpha}{w} \cdot \mu.$$

The function p^x has the property that $\Theta^G(x^*, p^x(w), w) = \Theta^F(x^*, w)$. Hence, $p^x(w^*) = p^*$.

Next, we define the auxiliary function $Z : (0, +\infty) \to (0, +\infty)$ by

$$Z(w) = f(x^*) \cdot a \cdot \alpha \cdot n^s(\Theta^F(x^*, w)).$$

Given that $\Theta^F(x^*, w^*) = \theta^*$ and Θ^F is strictly decreasing in w, $\Theta^F(x^*, w) \in [\theta^*, \theta^m)$ if $w \in (0, w^*]$ and $\Theta^F(x^*, w) \in (0, \theta^*]$ if $w \in [w^*, +\infty)$. Since n^s is strictly increasing on $[0, \theta^*]$ and strictly decreasing on $[\theta^*, \theta^m]$ and Θ^F is strictly decreasing in w, we infer that Z is strictly increasing for $w \in (0, w^*]$ and strictly decreasing for $w \in [w^*, +\infty)$. Since $n^s(0) = n^s(\theta^m) = 0$, $\lim_{w \to 0} \Theta^F(x^*, w) = \theta^m$, and $\lim_{w \to +\infty} \Theta^F(x^*, w) = 0$, we infer that $\lim_{w \to 0} Z(w) = 0$ and $\lim_{w \to +\infty} Z(w) = 0$.

The definition of Θ^F implies that $Z(w) = h \cdot w \cdot \hat{f}(\Theta^F(x^*, w))$. Thus,

$$p^x(w) = \frac{(1 + \tau(x^*))^{1-\varepsilon}}{Z(w)} \cdot \chi^{\varepsilon} \cdot \alpha \cdot \mu.$$

The properties of Z imply that the function p^x is strictly decreasing for $w \in (0, w^*]$ and strictly increasing for $w \in [w^*, +\infty)$, $\lim_{w \to 0} p^x(w) = +\infty$, and $\lim_{w \to +\infty} p^x(w) = +\infty$.

Last, we denote equilibrium labor market tightness by θ^e and equilibrium product market tightness by x^e. Consider $w \in (0, +\infty)$ and $p > p^x(w)$. Then $\Theta^G(x^*, p, w) < \Theta^G(x^*, p^x(w), w) = \Theta^F(x^*, w)$ because Θ^G is strictly decreasing in p. Given that Θ^F is strictly increasing in x and Θ^G is strictly decreasing in x and $\Theta^G(x^e, p, w) = \Theta^F(x^e, w)$, we conclude that $x^e < x^*$. In sum, for any $w > 0$ and $p > p^x(w)$, we have $x^e < x^*$. Similarly, for any $w > 0$ and $p < p^x(w)$, we have

$x^e > x^*$. $\square$

In Figure VII, the function p^θ is represented by the downward-sloping line and the function p^x is represented by the u-shaped line. The two curves intersect at (w^*, p^*).

Online Appendix C: Optimal Control Problems

This appendix solves the optimal control problems of the household and firm in the dynamic model of Section IV.

The Optimal Control Problem of the Household. Let $b(t) \equiv m(t)/p(t)$ denote real money balances. The law of motion of $b(t)$ is obtained from (13):

$$\dot{b}(t) = w(t) \cdot l(t) - y(t) - \pi \cdot b(t) + \frac{T(t)}{p(t)}$$

To solve the problem, we set up the current-value Hamiltonian

$$\begin{aligned} \mathscr{H}(t, c(t), y(t), b(t)) = & \frac{\chi}{1+\chi} \cdot c(t)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+\chi} \cdot b(t)^{\frac{\varepsilon-1}{\varepsilon}} + Y(t) \cdot \left[\frac{q(x(t))}{\rho} \cdot (y(t) - c(t)) - s \cdot y(t)\right] \\ & + B(t) \cdot \left[w(t) \cdot l(t) - y(t) - \pi \cdot b(t) + \frac{T(t)}{p(t)}\right] \end{aligned}$$

with control variable $c(t)$, state variables $y(t)$ and $b(t)$, and costate variables $Y(t)$ and $B(t)$.

The necessary conditions for an interior solution to this maximization problem are $\partial \mathscr{H} / \partial c(t) = 0$, $\partial \mathscr{H} / \partial y(t) = \delta \cdot Y(t) - \dot{y}(t)$, and $\partial \mathscr{H} / \partial b(t) = \delta \cdot B(t) - \dot{b}(t)$, together with the transversality conditions $\lim_{t \to +\infty} e^{-\delta \cdot t} \cdot Y(t) \cdot y(t) = 0$ and $\lim_{t \to +\infty} e^{-\delta \cdot t} \cdot B(t) \cdot b(t) = 0$. Given that $\mathscr{H}$ is concave in (c, y, b), these conditions are also sufficient.

These three conditions can be rewritten as

$$\begin{aligned} \frac{\chi}{1+\chi} \cdot \frac{\varepsilon - 1}{\varepsilon} \cdot c(t)^{\frac{-1}{\varepsilon}} &= Y(t) \cdot \frac{q(x(t))}{\rho} \\ Y(t) \cdot \left(\frac{q(x(t))}{\rho} - s\right) - B(t) &= \delta \cdot Y(t) - \dot{y}(t) \\ \frac{1}{1+\chi} \cdot \frac{\varepsilon - 1}{\varepsilon} \cdot b(t)^{\frac{-1}{\varepsilon}} - B(t) \cdot \pi &= \delta \cdot B(t) - \dot{b}(t) \end{aligned}$$

In steady state, $\dot{y}(t) = \dot{b}(t) = 0$. Hence, after eliminating the costate variables B and Y, we find that the optimal consumption decision of the household is

$$c = \chi^{\varepsilon} \cdot (\delta + \pi)^{\varepsilon} \cdot \left[1 - (\delta + s) \cdot \frac{\rho}{q(x)}\right]^{\varepsilon} \cdot b.$$

We obtain the equation in the text by setting $\delta = 0$ and $b = \mu(0)/p(0)$ in the above equation.

The Optimal Control Problem of the Firm. To solve this problem, we set up the current-value Hamiltonian

$$\begin{aligned}\mathscr{H}(t, n(t), y(t), l(t)) =& y(t) - w(t) \cdot l(t) + Y(t) \cdot [f(x(t)) \cdot (a \cdot n(t)^{\alpha} - y(t)) - s \cdot y(t)] \\ & + L(t) \cdot \left[\frac{\hat{q}(\theta(t))}{\hat{\rho}} \cdot (l(t) - n(t)) - \hat{s} \cdot l(t)\right]\end{aligned}$$

with control variable $n(t)$, state variables $y(t)$ and $l(t)$, and current-value costate variables $Y(t)$ and $L(t)$. The necessary conditions for an interior solution to this maximization problem are $\partial\mathscr{H}/\partial n(t) = 0$, $\partial\mathscr{H}/\partial y(t) = \delta \cdot Y(t) - \dot{y}(t)$, and $\partial\mathscr{H}/\partial l(t) = \delta \cdot L(t) - \dot{l}(t)$, together with the transversality conditions $\lim_{t\to+\infty} e^{-\delta \cdot t} \cdot Y(t) \cdot y(t) = 0$ and $\lim_{t\to+\infty} e^{-\delta \cdot t} \cdot L(t) \cdot l(t) = 0$. Given that $\mathscr{H}$ is concave in (n, y, l), these conditions are also sufficient.

These three conditions can be rewritten as

$$\begin{aligned} Y(t) \cdot f(x(t)) \cdot \alpha \cdot a \cdot n(t)^{\alpha-1} &= L(t) \cdot \frac{\hat{q}(\theta(t))}{\hat{\rho}} \\ 1 &= Y(t) \cdot (f(x(t)) + s) + \delta \cdot Y(t) - \dot{y}(t) \\ L(t) \cdot \left(\frac{\hat{q}(\theta(t))}{\hat{\rho}} - \hat{s}\right) &= w(t) + \delta \cdot L(t) - \dot{l}(t) \end{aligned}$$

In steady state, $\dot{l}(t) = \dot{y}(t) = 0$. Hence, after eliminating the costate variables L and Y, we find that the optimal employment decision of the firm satisfies

$$n = \left\{\frac{\alpha \cdot a}{w} \cdot \frac{f(x)}{\delta + s + f(x)} \cdot \left[1 - (\delta + \hat{s}) \cdot \frac{\hat{\rho}}{\hat{q}(\theta)}\right]\right\}^{\frac{1}{1-\alpha}}.$$

We obtain equation in the text by setting $\delta = 0$ in the above equation.

Online Appendix D: Another Proxy for Product Market Tightness

This appendix proposes another proxy for the cyclical component of the product market tightness, and it shows that all the empirical results are robust to using this alternative proxy. The proxy is constructed from the operating rate in non-manufacturing sectors measured by the Institute for Supply Management (ISM) and published in their Semiannual Reports. This operating rate is available for the 1999:Q4–2013:Q2 period. In the text, the proxy is constructed from the capacity utilization rate in the manufacturing sector measured by the Census Bureau from the Survey of Plant Capacity (SPC).

Using the operating rate from the ISM is conceptually better than using the capacity utilization rate from the SPC for two reasons. First, the operating rate is a direct measure of labor utilization; therefore, it is directly linked to product market tightness. Second, the operating rate applies to non-manufacturing sectors, where logistical issues such as peak load and inventory management do not influence labor utilization. We do not use this alternative proxy in the text, however, because it is only available for a brief period (1999:Q4–2013:Q2) that does not cover sufficiently many business cycles to permit a thorough empirical analysis.

We construct our alternative proxy for the cyclical component of the product market tightness as follows. The operating rate or_t measured by the ISM is the actual production level of firms as a share of their maximum production level given their current capital stock and workforce. Since the operating rate takes labor as a fixed factor, it exactly corresponds to our concept of labor utilization: $or_t = f(x_t)/(s + f(x(t)))$.[35] The ISM measures or_t in the second and fourth quarter; we use a linear interpolation of the biannual series to transform it into a quarterly series for the 1999:Q4–2013:Q2 period. Then, we remove from $\ln(or_t)$ the trend produced by a HP filter with smoothing parameter 1600. The resulting detrended series is our proxy for the cyclical component of the product market tightness. This proxy is plotted in Figure A2, together with the proxy used in the text. We refer to the proxy in the text as the *SPC proxy* and to this alternative proxy as the *ISM proxy*.

Over the 1999:Q4–2013:Q2 period, the correlation between the two proxies is 0.67. As showed in Figure A2, the two proxies behaved similarly over the period: both fell after 2001, picked up in

[35] This is a major difference with the capacity utilization rate from the SPC, which takes labor as a variable factor and thus requires a correction to be converted into a labor utilization rate. Morin and Stevens (2005) discuss the difference between the capacity utilization rate collected in the SPC and the operating rate collected in the ISM survey.

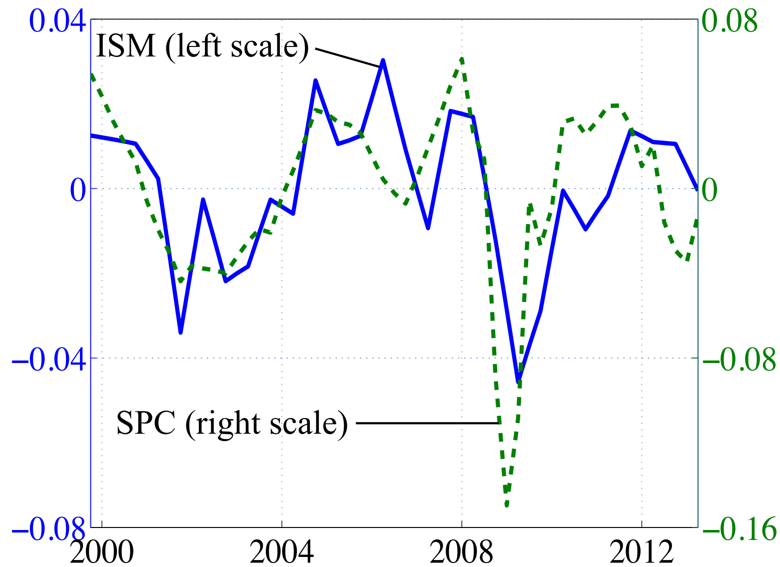


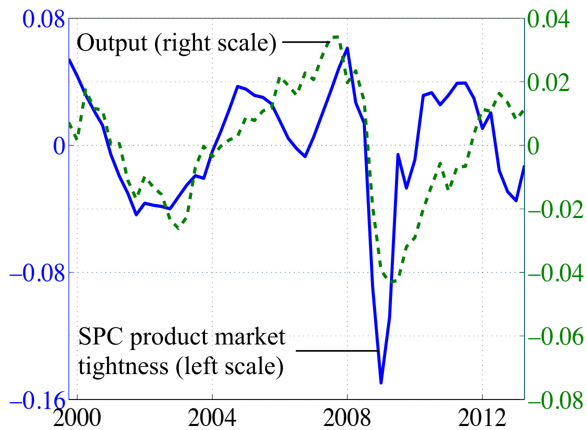
Figure A2: The Two Proxies for the Cyclical Component of Product Market Tightness

The time period is 1999:Q4–2013:Q2. The ISM proxy is constructed in Online Appendix D from the operating rate in non-manufacturing sectors constructed by the ISM. The SPC proxy is constructed in Section V from the capacity utilization rate in the manufacturing sector measured by the Census Bureau from the SPC.

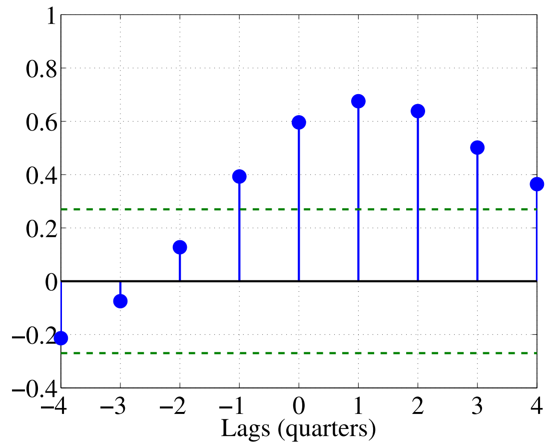
the 2004–2008 period, and collapsed in 2009, before recovering. The main difference is that the SPC proxy is subject to larger fluctuations than the ISM proxy.

In the text, we use the SPC proxy to identify aggregate demand and technology shocks. The finding reported in Figure X is that the SPC proxy and output are positively correlated, which implies that aggregate demand shocks are the main source of labor market fluctuations. Panels A and B of Figure A3 confirm that this result remains valid if we focus on the correlation between SPC proxy and output over the subperiod 1999:Q4–2013:Q2. The correlations between SPC proxy and output are slightly higher: at one lag, the correlation is 0.68; the contemporaneous correlation is 0.60. These correlations are statistically significant.

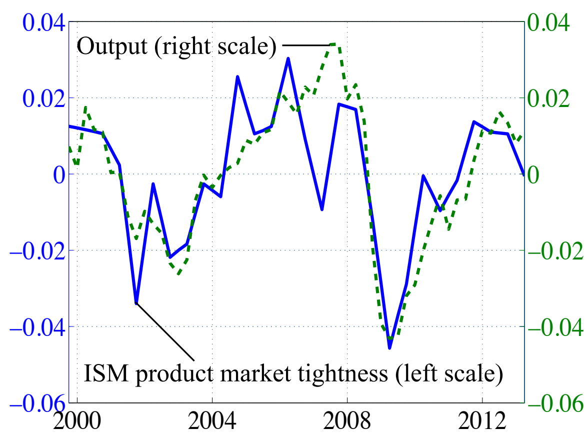
Panels C and D of Figure A3 show that we obtain the same result if we use the ISM proxy instead of the SPC proxy. The correlations of ISM proxy and output are even slightly higher than those of SPC proxy and output: the contemporaneous correlation is 0.82; at one lag, the correlation is 0.77. These correlations are statistically significant.



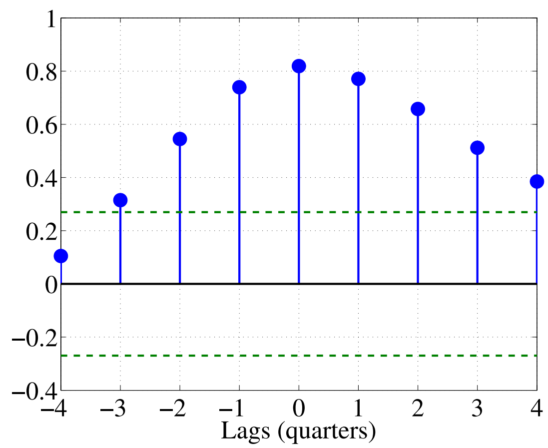
A. Cyclical components



B. Cross-correlogram (SPC tightness leads)



C. Cyclical components



D. Cross-correlogram (ISM tightness leads)

Figure A3: Correlation Between Product Market Tightness and Output

The time period is 1999:Q4–2013:Q2. Panel A displays the SPC proxy for the cyclical component of the product market tightness, $x_t^c(SPC)$, and the cyclical component of output, y_t^c. The construction of $x_t^c(SPC)$ is explained in Section V. Output, y_t, is the seasonally adjusted quarterly index for real output in the nonfarm business sector constructed by the BLS MSPC program. We construct y_t^c by removing from $\ln(y_t)$ the trend produced by a HP filter with smoothing parameter 1600. Panel B displays the cross-correlogram between $x_t^c(SPC)$ and y_t^c. The cross-correlation at lag i is the correlation between $x_{t-i}^c(SPC)$ and y_t^c. Panel C displays the ISM proxy for the cyclical component of the product market tightness, $x_t^c(ISM)$, and y_t^c. The ISM proxy is constructed in Online Appendix D. Panel D displays the cross-correlogram between $x_t^c(ISM)$ and y_t^c. The cross-correlation at lag i is the correlation between $x_{t-i}^c(ISM)$ and y_t^c. The horizontal dashed lines are the two-standard-deviation confidence bounds.

ONLINE APPENDIX E: ANOTHER TYPE OF MATCHING COST

This appendix proposes an alternative to the basic model of Section II in which the cost of matching is a time cost instead of an output cost. In this alternative model, households share their time between supplying services and matching with other households who sell services. In the original model, households spend all their time supplying services and they purchase services to match with other households. Yet, all the results of Section II remain valid in this alternative model.

Households employ their time to purchase services: the more time they spend on purchasing services, the less time they have to supply services. A visit takes away an amount $\rho > 0$ of the household's productive capacity; therefore, the actual productive capacity of a household making v visits is $k - \rho \cdot v$. The number of matches on the product market is

$$y = \left[(k - \rho \cdot v)^{-\gamma} + v^{-\gamma}\right]^{-\frac{1}{\gamma}},$$

and the product market tightness is defined by

$$x = \frac{v}{k - \rho \cdot v}.$$

The probability to sell one of the $k - \rho \cdot v$ services for sale is $f(x)$. The probability that a visit is successful is $q(x)$.

To purchase c services, household need to make $c/q(x)$ visits that take away an amount $\rho \cdot c/q(x)$ of their productive capacity. Households are left with a capacity $k - \rho \cdot c/q(x)$, and they sell a fraction $f(x)$ of it. Furthermore, output is equal to consumption—and welfare—because no part of output is used for matching. Therefore, $c = y = f(x) \cdot (k - \rho \cdot c/q(x))$. The aggregate supply is the amount of consumption that solves this equation. Since $f(x)/q(x) = x$, the aggregate supply admits the following expression:

$$c^s(x) = \frac{f(x)}{1 + \rho \cdot x} \cdot k.$$

The aggregate supply is strictly increasing for $x \in [0, x^*]$ and strictly decreasing for $x \in [x^*, +\infty)$

where x^* is the unique solution to

$$q(x)^\gamma = \frac{\rho \cdot x}{1+\rho \cdot x}.$$

This equation is obtained by rearranging $dc^s/dx = 0$ and using the fact that $f'(x) = q(x)^{1+\gamma}$. It admits a unique solution because q is strictly decreasing from 1 to 0 on $[0,+\infty)$ while $x \mapsto (\rho \cdot x)/(1+\rho \cdot x)$ is strictly increasing on 0 to 1 on $[0,+\infty)$. The tightness x^* is the efficient tightness: it maximizes welfare for a given level of real money balances. As in the original model, x^* depends only on the matching function and on the matching cost.

Since all output is used for consumption, there is no price wedge due to matching, and the price of consumption is p. However, the selling capacity of the household, and thus its income, is reduced because of the time spent buying consumption. The household's budget constraint is therefore modified to

$$m + p \cdot c = \mu + p \cdot f(x) \cdot \left(k - \rho \cdot \frac{c}{q(x)}\right).$$

To see the parallel between this budget constraint and the budget constraint in Section II, it is convenient to rewrite this constraint as

$$m + p \cdot (1+\rho \cdot x) \cdot c = \mu + p \cdot f(x) \cdot k.$$

From the household's perspective, the time required to buy consumption imposes a wedge $\rho \cdot x$ on the price of consumption. Accordingly, the household's optimal level of consumption is the same as in the model of Section II after replacing the old price wedge, $\tau(x)$, by the new price wedge, $\rho \cdot x$. The aggregate demand therefore is

$$c^d(x,p) = \left(\frac{\chi}{1+\rho \cdot x}\right)^\varepsilon \cdot \frac{\mu}{p}.$$

The aggregate demand is strictly decreasing in x and p for all $x > 0$ and $p > 0$.

Given that the aggregate supply and demand are isomorphic to those in the original model once $\tau(x)$ is replaced by $\rho \cdot x$, we can analyze this alternative model by following the same steps. We can show that all the properties of the original model carry over, with one exception. In a fixprice equilibrium, the comparative statics for output and consumption are the same, and they are the same as the comparative statics for consumption in the original model. But they are different from the comparative statics for output in the original model. In the slack regime this difference is mute because consumption and output move together in the original model. But in the tight regime this difference is visible: after an increase in aggregate demand, output increases in the original model but decreases in the alternative model.

We think that the original model of Section II is more realistic than this alternative model because the result that output is higher when the economy becomes tighter seems more realistic, at least for Western economies. The main difference between the original and the alternative model is that the resources devoted to matching are marketed in the original model but not in the alternative model. The alternative model perhaps describes better the centralized economies of the Soviet Union where fewer services were marketed. It is possible that in those economies, people spent so much time queuing to buy goods and services that they had to reduce their supply of labor, and output was lower than if aggregate demand had been lower.

Online Appendix F: Endogenous Marketing Effort

This appendix proposes an extension of the basic model of Section II in which households devote marketing effort to increase their sales. In this alternative model, households share their productive capacity between supplying services and marketing these services. In the original model, households spend all their productive capacity on supplying services. All the results of Section II remain valid in this extension.

Households spend an amount $a \leq k$ of their productive capacity on marketing. The amount of productive capacity left for supplying services is $k - a$. Marketing increases the visibility of services for sale and thus their probability of being sold. The function $e : [0, k] \to [0, 1]$ describes the effectiveness of marketing. We assume that e is strictly increasing and concave. To ensure an interior solution with positive marketing effort, we assume $e(0) = 0$. To simplify, we assume that the function e has a constant elasticity ε. As in Pissarides (2000, Chapter 5), the number of matches on the product market is given by

$$y = \left\{ [e(a) \cdot (k - a)]^{-\gamma} + v^{-\gamma} \right\}^{-\frac{1}{\gamma}},$$

and the product market tightness is defined by

$$x = \frac{v}{e(a) \cdot (k - a)}.$$

The probability to sell one service is $e(a) \cdot f(x)$. Hence, a higher amount of marketing generates more sales. The probability that a visit is successful is $q(x)$.

Households choose their marketing effort to maximize their income. Given x, they choose a to maximize $e(a) \cdot f(x) \cdot (k - a)$. The optimal a satisfies $\varepsilon = a/(k - a)$, which can be rewritten as

$$a = \frac{\varepsilon}{1 + \varepsilon} \cdot k.$$

It is optimal for households to devote a fraction $\varepsilon/(1 + \varepsilon)$ of their productive capacity to marketing.

The aggregate supply describes the amount of consumption sold given the matching process and the optimal marketing decision of households. The aggregate supply admits the following expression:

$$c^s(x) = (f(x) - \rho \cdot x) \cdot e\left(\frac{\varepsilon}{1 + \varepsilon} \cdot k\right) \cdot \frac{1}{1 + \varepsilon} \cdot k.$$

Although it admits a different expression, the aggregate supply has the same properties as in the model without marketing. Furthermore, the aggregate demand remains the same because the trade-off between consumption and holding money is not affected by the marketing effort.

We can analyze this extension with endogenous marketing effort as we analyzed the original model. Since aggregate demand and supply retain the properties of the original model, we can show that in fact all the properties of the original model carry over.

References

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