A Theory of Economic Slack

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APPENDIX J.

Policy multipliers in the two-market model

In this appendix, we compute policy multipliers in the two-market slackish business cycle model of chapter 15. We begin with the monetary multiplier, which describes the effect of a change in the nominal interest rate on the unemployment rate. We then turn to the fiscal multiplier, which gives the effect of a change in government spending on the unemployment rate.

J.1. Background

The key in computing the policy multipliers is to determine the response of labor market tightness to the policy change. From that it is simple to compute the response of other variables and eventually the multiplier. Given that labor market tightness is given by equation (15.20), we need to characterize the shapes of the curves $\theta_p^l(\theta_l, w)$ and $\theta_p^p(\theta_l, r, w)$. It is convenient to rewrite the definition of $\theta_p^l(\theta_l, w)$, given by equation (15.18), as follows:

$$1 - u_p(\theta_p^l) = \frac{wh^{\alpha_l}\left[1 - u_l(\theta_l)\right]^{\alpha_l}\left[1 + \tau_l(\theta_l)\right]^{1-\alpha_l}}{(1-\alpha_l)a_l}. \tag{J.1}$$

And it is convenient to rewrite the definition of $\theta_p^p(\theta_l, r, w)$, given by equation (15.19), as follows:

$$\left[1 + \tau_p(\theta_p^p)\right]^{1-\alpha_p} = \frac{(1-\alpha_l)^{\alpha_p}\left[(1-\alpha_p)(\delta - r)a_p\right]}{\left(wh\left[1 - u_l(\theta_l)\right]\right)^{\alpha_p}}. \tag{J.2}$$

To compute the multipliers, we will rely heavily on the elasticity results from appendix B. An elasticity that we will use repeatedly is the elasticity of $1+\tau(\theta)$ in any market, which is given by (6.12):

$$\epsilon_{\theta}^{1+\tau} = \eta(\theta)\tau(\theta). \tag{J.3}$$

Another useful elasticity is the elasticity of the slack rate $u(\theta)$ in any market, which is given by (13.13):

$$\epsilon_{\theta}^{u} = -\left[1-\eta(\theta)\right]\left[1-u(\theta)\right]. \tag{J.4}$$

A final useful elasticity is the elasticity of $1-u(\theta)$ in any market, which can be inferred from (13.13):

$$\epsilon_{\theta}^{1-u} = \frac{-u(\theta)}{1-u(\theta)} \cdot \epsilon_{\theta}^{u} = \left[1-\eta(\theta)\right]u(\theta). \tag{J.5}$$

With the appropriate matching wedge, matching elasticity, and slack rate, these results apply to both the product market and labor market.

J.2. Monetary multiplier

We are now ready to compute the monetary multiplier.

The solution of the two-market model is given by (15.20), which imposes $\theta_p^l(\theta_l, w) = \theta_p^p(\theta_l, r, w)$. We implicitly differentiate this equation in elasticity to obtain the elasticity of labor market tightness with respect to the real interest rate (result B.10). We get

$$\epsilon_r^{\theta_l} = \frac{\epsilon_r^{\theta_p^p}}{\epsilon_{\theta_l}^{\theta_p^l} - \epsilon_{\theta_l}^{\theta_p^p}}. \tag{J.6}$$

Hence, to obtain the response of labor market tightness to the real interest rate, we need to compute the three elasticities in the equation: $\epsilon_r^{\theta_p^p}$, $\epsilon_{\theta_l}^{\theta_p^l}$, and $\epsilon_{\theta_l}^{\theta_p^p}$.

First, we implicitly differentiate in elasticity the definition of θ_p^p given by (J.2) to obtain the elasticity of θ_p^p with respect to the real interest rate. We use (J.3) for the calculation. We get

$$\epsilon_r^{\theta_p^p} = \frac{-r/(\delta-r)}{(1-\alpha_p)\eta_p(\theta_p)\tau_p(\theta_p)}. \tag{J.7}$$

Second, we implicitly differentiate in elasticity the definition of θ_p^p given by (J.2) to obtain the elasticity of θ_p^p with respect to labor market tightness. We use both (J.3) and

(J.5) for the calculation. We get

$$\epsilon_{\theta_l}^{\theta_p^p} = \frac{-\alpha_p\left[1-\eta_l(\theta_l)\right]u_l(\theta_l)}{(1-\alpha_p)\eta_p(\theta_p)\tau_p(\theta_p)}. \tag{J.8}$$

Third, we implicitly differentiate in elasticity the definition of θ_p^l given by (J.1) to obtain the elasticity of θ_p^l with respect to labor market tightness. We use both (J.3) and (J.5) for the calculation. We get

$$\epsilon_{\theta_l}^{\theta_p^l} = \frac{\alpha_l\left[1-\eta_l(\theta_l)\right]u_l(\theta_l)+(1-\alpha_l)\eta_l(\theta_l)\tau_l(\theta_l)}{\left[1-\eta_p(\theta_p)\right]u_p(\theta_p)}. \tag{J.9}$$

We now combine equations (J.6), (J.7), (J.8), and (J.9) to compute the elasticity of labor market tightness with respect to the real interest rate. The raw expression for the elasticity is:

$$\epsilon_r^{\theta_l} = \frac{\frac{-r}{\delta-r}\cdot\frac{1}{(1-\alpha_p)\eta_p(\theta_p)\tau_p(\theta_p)}}{\frac{\alpha_l[1-\eta_l(\theta_l)]u_l(\theta_l)+(1-\alpha_l)\eta_l(\theta_l)\tau_l(\theta_l)}{[1-\eta_p(\theta_p)]u_p(\theta_p)}+\frac{\alpha_p[1-\eta_l(\theta_l)]u_l(\theta_l)}{(1-\alpha_p)\eta_p(\theta_p)\tau_p(\theta_p)}}.$$

After some simplification, we get:

$$\epsilon_r^{\theta_l} = \frac{-r}{\delta-r}\cdot\frac{1}{[1-\eta_l(\theta_l)]u_l(\theta_l)}\cdot\frac{1}{(1-\alpha_p)\left[\alpha_l+(1-\alpha_l)\frac{\eta_l(\theta_l)\tau_l(\theta_l)}{[1-\eta_l(\theta_l)]u_l(\theta_l)}\right]\frac{\eta_p(\theta_p)\tau_p(\theta_p)}{[1-\eta_p(\theta_p)]u_p(\theta_p)}+\alpha_p}.$$

Then, given that the unemployment rate is given by $u_l(\theta_l)$, with an elasticity (J.4), the elasticity of the unemployment rate with respect to the real interest rate is given by $\epsilon_r^{u_l} = -[1-\eta_l(\theta_l)][1-u_l(\theta_l)]\epsilon_r^{\theta_l}$, which simplifies to:

$$\epsilon_r^{u_l} = \frac{r}{\delta-r}\cdot\frac{1-u_l(\theta_l)}{u_l(\theta_l)}\cdot\frac{1}{(1-\alpha_p)\left[\alpha_l+(1-\alpha_l)\frac{\eta_l(\theta_l)\tau_l(\theta_l)}{[1-\eta_l(\theta_l)]u_l(\theta_l)}\right]\frac{\eta_p(\theta_p)\tau_p(\theta_p)}{[1-\eta_p(\theta_p)]u_p(\theta_p)}+\alpha_p}. \tag{J.10}$$

Accordingly, the monetary multiplier $du_l/di = du_l/dr = (u_l(\theta_l)/r)\cdot\epsilon_r^{u_l}$ is given by:

$$\frac{du_l}{di} = \frac{1-u_l(\theta_l)}{\delta-r}\cdot\frac{1}{\alpha_p+(1-\alpha_p)\left[\alpha_l+(1-\alpha_l)\frac{\eta_l(\theta_l)\tau_l(\theta_l)}{[1-\eta_l(\theta_l)]u_l(\theta_l)}\right]\frac{\eta_p(\theta_p)\tau_p(\theta_p)}{[1-\eta_p(\theta_p)]u_p(\theta_p)}}. \tag{J.11}$$

J.3. Fiscal multiplier

Next we turn to the fiscal multiplier. We are interested in the effect of government spending on the unemployment rate. We express government spending as a share of private spending, so the aggregate demand with fiscal policy is

$$(1+g)\cdot y^d(\theta_p, r),$$

where $y^d(\theta_p, r)$ is the private demand given by (15.11). Expressing government spending in this fashion greatly simplifies the computation. It is also consistent with the way government spending appears in the analysis of optimal fiscal policy of chapter 17: what matters for welfare is the ratio between public and private consumption, which corresponds to g.

The term $1+g$ from the aggregate demand then appears in equation (J.2), which becomes:

$$\left[1+\tau_p(\theta_p^p)\right]^{1-\alpha_p} = \frac{(1+g)^{\alpha_p}(1-\alpha_l)^{\alpha_p}\left[(1-\alpha_p)(\delta-r)a_p\right]}{\left(wh\left[1-u_l(\theta_l)\right]\right)^{\alpha_p}}. \tag{J.12}$$

Equation (J.1) on the other hand is unchanged.

We compute the fiscal multiplier just like the monetary multiplier, following the same steps. First, we implicitly differentiate equation (15.20), where $\theta_p^p(\theta_l, r, w, g)$ is now given by (J.12), in elasticity to obtain the elasticity of labor market tightness with respect to government spending. As above, we get

$$\epsilon_g^{\theta_l} = \frac{\epsilon_g^{\theta_p^p}}{\epsilon_{\theta_l}^{\theta_p^l} - \epsilon_{\theta_l}^{\theta_p^p}}. \tag{J.13}$$

The elasticities $\epsilon_{\theta_l}^{\theta_p^l}$ and $\epsilon_{\theta_l}^{\theta_p^p}$ remain the same as above: they are not affected by fiscal policy. The elasticity $\epsilon_g^{\theta_p^p}$ is simply obtained by implicitly differentiating in elasticity the definition of θ_p^p given by (J.12). We get

$$\epsilon_g^{\theta_p^p} = \frac{\alpha_p g/(1+g)}{(1-\alpha_p)\eta_p(\theta_p)\tau_p(\theta_p)}. \tag{J.14}$$

The elasticity is fairly similar to that in (J.7): the sole difference is that $-r/(\delta-r)$ is replaced by $\alpha_p g/(1+g)$.

Hence, we can compute the elasticities $\epsilon_g^{\theta_l}$ and $\epsilon_g^{u_l}$ just as in the case of monetary policy, but replacing $-r/(\delta-r)$ by $\alpha_p g/(1+g)$. Based on (J.10), this gives

$$\epsilon_g^{u_l} = \frac{-g}{1+g}\cdot\frac{1-u_l(\theta_l)}{u_l(\theta_l)}\cdot\frac{1}{1+\frac{1-\alpha_p}{\alpha_p}\left[\alpha_l+(1-\alpha_l)\frac{\eta_l(\theta_l)\tau_l(\theta_l)}{[1-\eta_l(\theta_l)]u_l(\theta_l)}\right]\frac{\eta_p(\theta_p)\tau_p(\theta_p)}{[1-\eta_p(\theta_p)]u_p(\theta_p)}}.$$

We normalize the fiscal multiplier to be positive, so we define the multiplier by $-du_l/dg = -[u_l(\theta_l)/g]\cdot\epsilon_g^{u_l}$. Given the expression above for the elasticity $\epsilon_g^{u_l}$, the fiscal multiplier is:

$$-\frac{du_l}{dg} = \frac{1-u_l(\theta_l)}{1+g}\cdot\frac{1}{1+\frac{1-\alpha_p}{\alpha_p}\left[\alpha_l+(1-\alpha_l)\frac{\eta_l(\theta_l)\tau_l(\theta_l)}{[1-\eta_l(\theta_l)]u_l(\theta_l)}\right]\frac{\eta_p(\theta_p)\tau_p(\theta_p)}{[1-\eta_p(\theta_p)]u_p(\theta_p)}}. \tag{J.15}$$