A Theory of Economic Slack

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APPENDIX I.

Policy multipliers in the two-market model

In this appendix, we compute policy multipliers in the two-market slackish business cycle model of chapter 15. We focus on the monetary multiplier, which describes the effect of a change in the nominal interest rate on the unemployment rate.

I.1. Background

The key in computing the policy multipliers is to determine the response of labor market tightness to the policy change. From that it is simple to compute the response of other variables and eventually the multiplier. Given that labor market tightness is given by equation (15.20), we need to characterize the shapes of the curves $\theta_p^l(\theta_l, w)$ and $\theta_p^p(\theta_l, r, w)$, defined by equations (15.18) and (15.19). It is convenient to rewrite the definition of $\theta_p^l(\theta_l, w)$ as follows:

$$1 - u_p(\theta_p^l) = \frac{wh^{\alpha_l}}{(1-\alpha_l)a_l}\left[1 - u_l(\theta_l)\right]^{\alpha_l}\left[1 + \tau_l(\theta_l)\right]^{1-\alpha_l}. \tag{I.1}$$

And it is convenient to rewrite the definition of $\theta_p^p(\theta_l, r, w)$ as follows:

$$1 + \tau_p(\theta_p^p) = \frac{(1-\alpha_l)\left[(1-\alpha_p)(\delta - r)a_p\right]^{1/(1-\alpha_p)}}{(wh)^{\alpha_p/(1-\alpha_p)}} \cdot \frac{1}{\left[1 - u_l(\theta_l)\right]^{\alpha_p/(1-\alpha_p)}}. \tag{I.2}$$

To compute the multipliers, we will rely heavily on the elasticity results from appendix B. An elasticity that we will use repeatedly is the elasticity of $1 + \tau(\theta)$ in any market, which is given by (6.12):

$$\epsilon_{\theta}^{1+\tau} = \eta(\theta)\tau(\theta). \tag{I.3}$$

Another useful elasticity is the elasticity of the slack rate $u(\theta)$ in any market, which is given by (13.13):

$$\epsilon_{\theta}^{u} = -\left[1 - \eta(\theta)\right]\left[1 - u(\theta)\right]. \tag{I.4}$$

A final useful elasticity is the elasticity of $1 - u(\theta)$ in any market, which can be inferred from (13.13):

$$\epsilon_{\theta}^{1-u} = \frac{-u(\theta)}{1 - u(\theta)} \cdot \epsilon_{\theta}^{u} = \left[1 - \eta(\theta)\right] u(\theta). \tag{I.5}$$

With the appropriate matching wedge, matching elasticity, and slack rate, these results apply to both the product market and labor market.

I.2. Monetary multiplier

We are now ready to compute the monetary multiplier.

First, the solution of the two-market model is given by (15.20), which imposes $\theta_p^l(\theta_l, w) = \theta_p^p(\theta_l, r, w)$. We implicitly differentiate this equation in elasticity to obtain the elasticity of labor market tightness with respect to the real interest rate (result B.10). We get

$$\epsilon_r^{\theta_l} = \frac{\epsilon_r^{\theta_p^p}}{\epsilon_{\theta_l}^{\theta_p^l} - \epsilon_{\theta_l}^{\theta_p^p}}. \tag{I.6}$$

Hence, to obtain the response of labor market tightness to the real interest rate, we need to compute the three elasticities in the equation: $\epsilon_r^{\theta_p^p}$, $\epsilon_{\theta_l}^{\theta_p^l}$, and $\epsilon_{\theta_l}^{\theta_p^p}$.

First, we implicitly differentiate in elasticity the definition of θ_p^p given by (I.2) to obtain the elasticity of θ_p^p with respect to the real interest rate. We use (I.3) for the calculation. We get

$$\epsilon_r^{\theta_p^p} = \frac{-r}{\delta - r} \cdot \frac{1}{1 - \alpha_p} \cdot \frac{1}{\eta_p(\theta_p)\tau_p(\theta_p)}. \tag{I.7}$$

Second, we implicitly differentiate in elasticity the definition of θ_p^p given by (I.2) to obtain the elasticity of θ_p^p with respect to labor market tightness. We use both (I.3) and

(I.5) for the calculation. We get

$$\epsilon_{\theta_l}^{\theta_p^p} = \frac{-\alpha_p}{1-\alpha_p} \cdot \frac{[1-\eta_l(\theta_l)]\, u_l(\theta_l)}{\eta_p(\theta_p)\tau_p(\theta_p)}. \tag{I.8}$$

Third, we implicitly differentiate in elasticity the definition of θ_p^l given by (I.1) to obtain the elasticity of θ_p^l with respect to labor market tightness. We use both (I.3) and (I.5) for the calculation. We get

$$\epsilon_{\theta_l}^{\theta_p^l} = \frac{\alpha_l\,[1-\eta_l(\theta_l)]\, u_l(\theta_l) + (1-\alpha_l)\eta_l(\theta_l)\tau_l(\theta_l)}{\left[1-\eta_p(\theta_p)\right] u_p(\theta_p)}. \tag{I.9}$$

We now combine equations (I.6), (I.7), (I.8), and (I.9) to compute the elasticity of labor market tightness with respect to the real interest rate. The raw expression for the elasticity is:

$$\epsilon_r^{\theta_l} = \frac{\frac{-r}{\delta-r} \cdot \frac{1}{1-\alpha_p} \cdot \frac{1}{\eta_p(\theta_p)\tau_p(\theta_p)}}{\frac{\alpha_l[1-\eta_l(\theta_l)]u_l(\theta_l)+(1-\alpha_l)\eta_l(\theta_l)\tau_l(\theta_l)}{[1-\eta_p(\theta_p)]u_p(\theta_p)} - \frac{-\alpha_p}{1-\alpha_p} \cdot \frac{[1-\eta_l(\theta_l)]u_l(\theta_l)}{\eta_p(\theta_p)\tau_p(\theta_p)}}.$$

After some simplification, we get:

$$\epsilon_r^{\theta_l} = \frac{1}{(1-\eta_l(\theta_l))u_l(\theta_l)} \cdot \frac{\frac{-r}{\delta-r}}{(1-\alpha_p)\left[\alpha_l + (1-\alpha_l)\frac{\eta_l(\theta_l)\tau_l(\theta_l)}{(1-\eta_l(\theta_l))u_l(\theta_l)}\right]\frac{\eta_p(\theta_p)\tau_p(\theta_p)}{(1-\eta_p(\theta_p))u_p(\theta_p)} + \alpha_p}.$$

Then, given that the unemployment rate is given by $u_l(\theta_l)$, with an elasticity (I.4), the elasticity of the unemployment rate with respect to the real interest rate is given by $\epsilon_r^{u_l} = -(1-\eta_l(\theta_l))(1-u_l(\theta_l))\epsilon_r^{\theta_l}$, which simplifies to:

$$\epsilon_r^{u_l} = \frac{(1-u_l(\theta_l))r}{(\delta-r)u_l(\theta_l)} \cdot \frac{1}{(1-\alpha_p)\left[\alpha_l + (1-\alpha_l)\frac{\eta_l(\theta_l)\tau_l(\theta_l)}{(1-\eta_l(\theta_l))u_l(\theta_l)}\right]\frac{\eta_p(\theta_p)\tau_p(\theta_p)}{(1-\eta_p(\theta_p))u_p(\theta_p)} + \alpha_p}.$$

Accordingly, the monetary multiplier $du_l/di = du_l/dr = (u_l(\theta_l)/r) \cdot \epsilon_r^{u_l}$ is given by:

$$\frac{du_l}{di} = \frac{1-u_l(\theta_l)}{\delta-r} \cdot \frac{1}{\alpha_p + (1-\alpha_p)\left[\alpha_l + (1-\alpha_l)\frac{\eta_l(\theta_l)\tau_l(\theta_l)}{(1-\eta_l(\theta_l))u_l(\theta_l)}\right]\frac{\eta_p(\theta_p)\tau_p(\theta_p)}{(1-\eta_p(\theta_p))u_p(\theta_p)}}.$$