A Theory of Slack

How Economic Slack Shapes Markets, Business Cycles, and Policies

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APPENDIX E.

Solution concept

This appendix discusses the solution concept from the slackish market model of chapter 5.

E.1. How much rationality does the model assume?

Let us now briefly discuss the amount of rationality assumed in the model. Economic models are often criticized for assuming too much rationality from people. The rational-expectations model, for example, assumes that people have a complete understanding of the structure of the economy and are able to predict how aggregate variables are likely to behave in the future. This is often seen as a level of rationality that is so high that it is unlikely to be met by actual people. Although our model is static, it is worth going through the different layers of rationality that are assumed in order to convince ourselves that the amount of rationality in the model is reasonable.

The first assumption is that buyers maximize their utility subject to their budget constraint. This means that they try to make the best decisions possible given their wealth. While it may not be realistic to expect buyers to be able to solve this type of maximization problem exactly, it is reasonable to assume that they understand their budget and the trade-off between consumption and saving.

A second assumption is that buyers anticipate prices correctly. This seems realistic because buyers know what they have paid in the past for similar goods and therefore should be able to anticipate the prices they will pay in the future. This doesn't require too much rationality because prices follow price norms that are, by definition, understood and followed by everyone in the market.

Last, we assume that buyers and sellers anticipate market tightness correctly. This is more complicated because tightness depends on the behavior of all sellers and buyers. Anticipating market tightness correctly is therefore quite difficult, as it requires people to know, for instance, the total quantity of goods offered by all sellers. With utility functions that are not quasilinear, buyers of different wealth would consume and shop differently, requiring an individual to know all of these variables to predict tightness. In that case anticipating tightness might be an unrealistic assumption.

Anticipating tightness is very similar to anticipating traffic. It requires anticipating the behavior of a large number of people who are moved by similar motives, accounting for random disturbances. People are typically good at predicting traffic on their regular itineraries, so they should be good at predicting tightness on familiar markets. What might be more difficult is predicting tightness on markets that they are unfamiliar with—just like it's difficult to predict traffic on unfamiliar routes. And just like it's hard to predict traffic after big accidents, it might be hard to predict tightness after big shocks.

E.2. Forecasting tightness with a statistical agency

Assuming that buyers and sellers anticipate market tightness correctly might be unrealistic. An alternative assumption that might be more realistic—and that does not change how the market operates—is that a statistical agency announces a correct forecast of market tightness at the beginning of the period. Buyers and sellers take this announcement at face value and behave optimally based on it. The key assumption here is that the statistical agency is able to make an accurate forecast, so the announced tightness ends up being the realized tightness.

The agency's goal is to quote an accurate market tightness to buyers and sellers. Market participants take this quote as given and make decisions to maximize their utility based on it. The realized tightness is equal to the quoted tightness if the agency makes a correct forecast. Let us now discuss how a statistical agency can forecast the correct tightness.

Let's assume that the statistical agency quotes a market tightness θ^a which people take as given. Then, buyer j aims to buy $y_j(\theta^a, p) = y^d(\theta^a, p)$, since the market demand corresponds to the individual demand for goods in the model. The buyers therefore visit $v_j(\theta^a, p) = y_j(\theta^a, p)/q(\theta^a) = y^d(\theta^a, p)/q(\theta^a)$ sellers. As a result, the realized tightness θ is:

$$\theta = \frac{\int_0^1 v_j(\theta^a, p)\, dj}{\int_0^1 k_i\, di} = \frac{y^d(\theta^a, p)}{q(\theta^a)k} = \theta^a \cdot \frac{y^d(\theta^a, p)}{\theta^a q(\theta^a)k} = \theta^a \cdot \frac{y^d(\theta^a, p)}{y^s(\theta^a)}.$$

Since the statistical agency aims to make a correct forecast, they are looking for θ^a

such that $\theta^a = \theta$. Therefore, they announce θ^a such that

$$y^s(\theta^a) = y^d(\theta^a, p).$$

In other words, the statistical agency announces the tightness θ^a that equalizes market demand and supply! Since the realized tightness θ equals the announced tightness θ^a, then the realized tightness also equalizes market demand and supply: $y^s(\theta) = y^d(\theta, p)$.

This model is a little more appealing because it requires much less rationality from people. The only thing they have to do is listen to the statistical agency and do the best they can given the tightness that has been announced. If they do that, and the agency announces the tightness equalizing market supply and demand, then the prevailing tightness is the announced tightness. So the agency is correct, and the supply-equals-demand tightness prevails. This model does require the agency to solve the supply-equals-demand equation, but a statistical agency is better equipped than laypeople to understand market conditions and solve the equation.

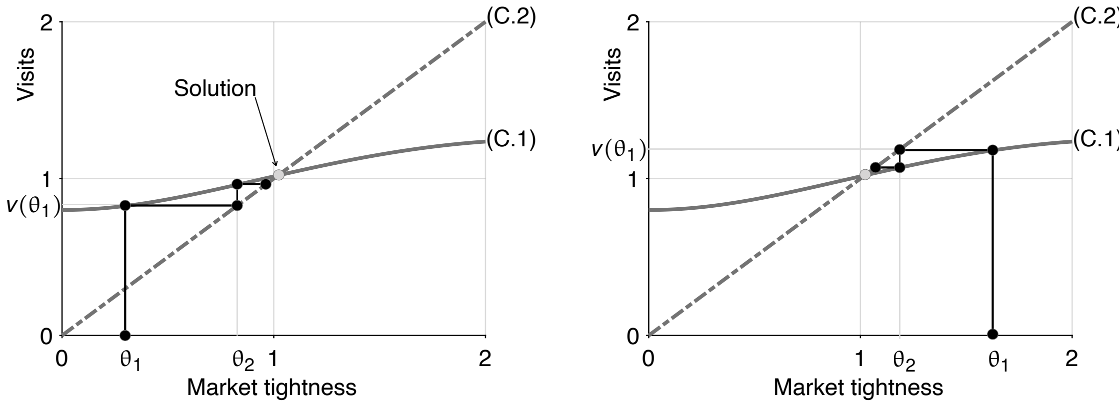
E.3. Convergence to the solution by tatonnement

Finally, what if the statistical agency cannot compute market demand and supply to forecast tightness? If computing this fixed point is difficult, the agency may attempt to learn it through an iterative process. Here we consider the tatonnement process through which the agency might converge to the correct forecast.

We imagine that the agency quotes a market tightness and asks buyers and sellers to report their plans under that tightness. Then the agency computes the tightness resulting from these plans. If the resulting tightness is not the same as the quoted tightness—which means that the agency would have misforecast tightness—the agency then re-quotes tightness, using the tightness that it has just computed from the plans they received. Then it asks buyers and sellers to report their new plans under the new tightness. The agency recomputes the tightness and checks whether it is the same as the quoted tightness. And so on, until the quoted and realized tightnesses converge.

The tatonnement process characterizes the evolution of tightness across iterations and shows whether the quoted tightness indeed converges to the model solution, which is given at the intersection of market supply and demand. Will the statistical agency eventually be able to quote the correct tightness, as a result of the tatonnement process?

Let's start from the first round of tatonnement. The statistical agency announces a tightness θ_1 and asks buyers and sellers to report their plans under that tightness. Sellers' plans do not depend on tightness, so they report their capacities k_i, which sum to $k = \int_0^1 k_i \, di$. We have seen in appendix D that if buyers believe that they will face a market tightness θ_1, the optimal number of visits for any buyer is $v(\theta_1)$, where $v(\theta)$ is defined by



A. Convergence to the solution from below

B. Convergence to the solution from above

FIGURE E.1. Numerical illustration of convergence by tatonnement

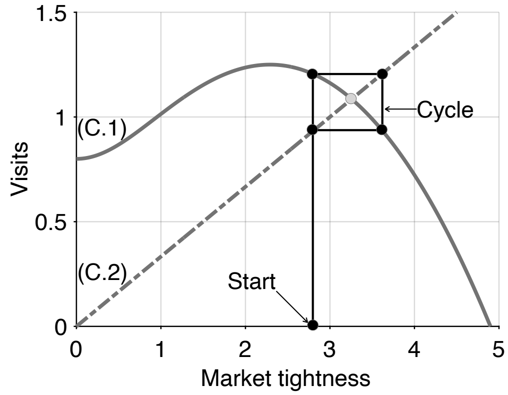
Equation (D.1) gives the optimal number of visits for buyers as a function of tightness. Equation (D.2) gives the number of visits obtained from the definition of market tightness. The matching function is CES, given by (4.11). Parameters are set to $\sigma = 2$, $\kappa = 0.2$, $k = 1$, $\alpha = 0.5$, and $a = 2$.

(D.1). Hence the aggregate number of visits is $v = \int_0^1 v(\theta_1)\, dj = v(\theta_1)$.

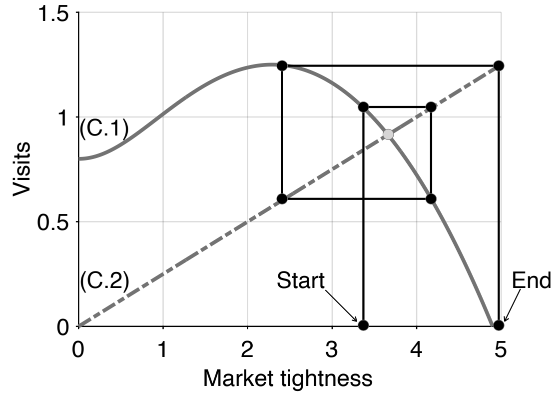
From the initial announcement θ_1, and the plans received from market participants, the agency then computes $\theta_2 = v(\theta_1)/k$. Are the tightnesses θ_2 and θ_1 the same? Well, it depends on the value of θ_1. Figure E.1A illustrates the tatonnement process when the agency announces a tightness that is too low, in the sense that it is below the solution θ of the model, which is at the intersection of the two curves. The agency announces θ_1, which we place on the x-axis. The optimal behavior by buyers if they believe that tightness will be θ_1 is to visit $v(\theta_1)$ sellers, as we established in appendix D. Receiving the plans of $v(\theta_1)$ visits, the agency computes a tightness of $\theta_2 = v(\theta_1)/k$, which can be obtained by projecting $v(\theta_1)$ on the line $v = \theta k$. We also place θ_2 on the x-axis and see that it is larger than θ_1, but still below the solution θ.

The next step for the agency is to announce θ_2. Following the same steps, buyers report a plan to visit $v(\theta_2)$ sellers, and the agency computes a new tightness $\theta_3 = v(\theta_2)/k$. Eventually, after many steps, the agency will approach the solution of the model, θ, which is at the intersection of the two curves. We can see this process visually in figure E.1, both when the agency starts from a tightness that is too low and when it starts with a tightness that is too high.

In the numerical case considered in figure E.1, the curve (D.1) is upward-sloping when it crosses the curve (D.2), so we can be sure that the tatonnement process converges to the model solution. But because it is difficult to characterize the properties of the curve (D.1) (as discussed in appendix D), it is difficult to guarantee that the tatonnement process always reaches the model solution. The curve (D.1) could be non-monotonic, and because of this the tatonnement process might not converge to the model solution.



A. Infinite cycle



B. Divergence to an infeasible tightness

FIGURE E.2. Failures of the tatonnement process to converge

Equation (D.1) gives the optimal number of visits for buyers as a function of tightness. Equation (D.2) gives the number of visits obtained from the definition of market tightness. The matching function is CES, given by (4.11). Parameters are set to $\sigma = 2$, $\kappa = 0.2$, $\alpha = 0.5$, and $a = 2$; in panel A, $k = 1/3$; in panel B, $k = 1/4$.

If the curve (D.1) is downward-sloping when it crosses (D.2), it is possible to construct examples where the tatonnement process does not converge. In figure E.2A, for instance, the tatonnement process cycles indefinitely, without ever converging. This is the type of pathology that Scarf (1960) had identified in the case of Walrasian models—showing that while the Walrasian model always has a unique solution, the tatonnement process might not converge to it. In figure E.2B, things are even worse: the tatonnement process rapidly leads the statistical agency to quote an infeasible tightness—a tightness so high that buyers do not want to enter the market at all (a θ such that (D.1) does not produce a positive v).

Hence, just like in Walrasian models, the tatonnement process is not guaranteed to converge to the solution in slackish models. An advantage of slackish models, however, is that the tatonnement process is completely natural. Once the statistical agency announces a tightness, market participants make decisions and report them, from which the agency can compute the realized tightness. This is quite different from the Walrasian tatonnement. There the auctioneer quotes a price, participants report the quantities that they would like to buy and sell, and if the quantities demanded and supplied are not the same, the auctioneer must quote a new price. Scarf (1960) assumes that the quoted price is adjusted by an amount commensurate to the excess market demand. But many other assumptions can be made, because the auctioneer may quote any price. In slackish models, the agency makes up a tightness only in the initial round. It then uses participants' plans to recompute tightnesses. This property might help determine formal conditions under which the tatonnement process is guaranteed to converge in slackish markets.

Bibliography

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