A Theory of Slack

How Economic Slack Shapes Markets, Business Cycles, and Policies

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APPENDIX C.

Visit-based solution

In chapter 5, we focused on a buyers' decisions about how much to consume and purchase. But in many models of search and matching, the key decision is framed differently: it's about the effort someone puts into searching. In our model, this effort is the number of visits to sellers.

Buyers' decisions on how many sellers to visit determines what they buy, what they spend, and what they save. So, it's useful to see how the model looks from this more traditional perspective. We now recast the model in terms of visits, solve it, and show it produces identical results. This exercise is instructive for various reasons. In particular, it reveals why our original output-based approach is more convenient than a visit-based approach.

C.1. Recasting the model in terms of visits

The first step is to rephrase the buyer's choice as a decision about visits (v_j) instead of consumption. This is straightforward because a buyer's consumption is directly proportional to their visits:

$$c_j = y_j - \kappa v_j = [q(\theta) - \kappa] \, v_j.$$

Buyer j chooses the number of visits v_j to maximize utility subject to their budget constraint, taking as given tightness and price. Thus, buyer j's problem becomes:

$$\max_{v_j \geq 0} a \, [q(\theta) - \kappa]^{1-\alpha} \, v_j^{1-\alpha} + b_j,$$

subject to the budget constraint:

$$b_j + pq(\theta)v_j = B_j.$$

Using the budget constraint to substitute money balances b_j out of the utility function, we rewrite the buyer's problem as

$$\max_{v_j \geq 0} a\left[q(\theta) - \kappa\right]^{1-\alpha} v_j^{1-\alpha} + B_j - pq(\theta)v_j.$$

This is just a concave maximization problem, so it suffices to take the first-order condition with respect to the number of visits to get the global maximum. The first-order condition gives:

$$(1-\alpha)a\left[q(\theta) - \kappa\right]^{1-\alpha} v_j^{-\alpha} = pq(\theta).$$

Solving for the optimal number of visits for buyer j yields:

$$v_j = \left[\frac{(1-\alpha)a}{pq(\theta)}\right]^{1/\alpha} \left[q(\theta) - \kappa\right]^{1/\alpha - 1}.$$

Then, the market-level number of visits is $v = \int_0^1 v_j \, dj$. Since all buyers visit the same number of sellers, and there is a mass 1 of buyers, the market-level number of visits is the same as the individual number of visits:

$$v = \left[\frac{(1-\alpha)a}{pq(\theta)}\right]^{1/\alpha} \left[q(\theta) - \kappa\right]^{1/\alpha - 1}. \tag{C.1}$$

Equation (C.1) is equivalent to the market demand that we derived in chapter 5. Thinking in terms of visits is therefore isomorphic to thinking in terms of output. Indeed, if we multiply both sides of (C.1) by $q(\theta)$, we get

$$q(\theta)v = \left[\frac{(1-\alpha)a}{p}\right]^{1/\alpha} \left[\frac{q(\theta) - \kappa}{q(\theta)}\right]^{1/\alpha - 1}.$$

Then we know that output satisfies $y = q(\theta)v$, via the matching function (5.1). By definition of the matching wedge (5.9), we also have

$$1 + \tau(\theta) = \frac{q(\theta)}{q(\theta) - \kappa}.$$

Thus, the equation can be rewritten

$$y = \left[\frac{(1-\alpha)a}{p}\right]^{1/\alpha} \left[1 + \tau(\theta)\right]^{1 - 1/\alpha}.$$

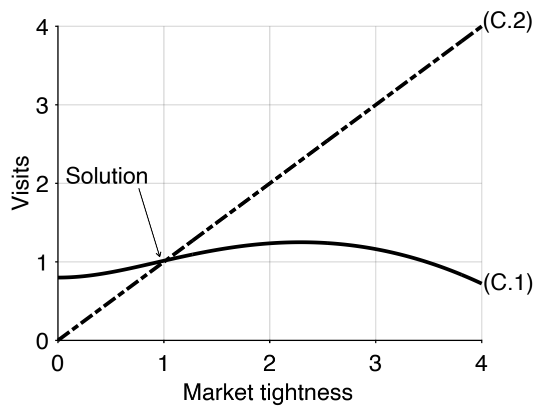

FIGURE C.1. Numerical illustration of the visit-based solution

Equation (C.1) gives the optimal number of visits for buyers as a function of tightness. Equation (C.2) gives the number of visits obtained from the definition of market tightness. The matching function is CES, given by (4.10). Parameters are set to $\sigma = 2$, $\kappa = 0.2$, $k = 1$, $\alpha = 0.5$, and $a = 2$.

Here we have just recovered that $y = y^d(\theta, p)$ where the market demand curve is given by (5.16). Hence, focusing on the number of visits instead of consumption does not change the market demand.

C.2. Solving the model in terms of visits

Our next step is to solve the model by determining the aggregate number of visits and tightness. The solution is equivalent to what we obtained before; however, we learn some interesting things from this visit-based approach.

To find tightness θ and visits v, we use two key equations linking tightness θ to visits v. The first is the demand equation (C.1). The second is the definition of tightness, $\theta = v/k$, which gives

$$v = \theta k. \tag{C.2}$$

Note that this equation is equivalent to the market supply, given by (5.6). Multiplying (C.2) by $q(\theta)$ gives $vq(\theta) = \theta q(\theta)k$, which is just $y = f(\theta)k$, or $y = y^s(\theta)$. Hence, focusing on the number of visits instead of output does not change the market supply.

The solution of the model must satisfy the two equations (C.1) and (C.2). Since equations (C.1) and (C.2) are equivalent to equations (5.6) and (5.16), the visit-based and output-based solutions are equivalent. We can plot these on a graph to help us visualize everything better (figure C.1). The solution (θ, v) is at the intersection of the two curves. From there we could determine all the other variables in the model, just as in the standard representation of the model in chapter 5.

Nevertheless, figure C.1 shows why the visit-based solution is not as analytically convenient as the output-based solution. We see that the curve $v(\theta)$ defined by equation (C.1) is not monotonic, and it is neither concave nor convex. Hence, establishing that it crosses the curve defined by equation (C.2) once and only once would not be trivial. Performing comparative statics would also be more complicated. By contrast, the output supply and demand curves (y^s and y^d) are well-behaved, making the analysis of the solution and its properties much more straightforward.