A Theory of Slack

How Economic Slack Shapes Markets, Business Cycles, and Policies

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APPENDIX A.

Hodrick-Prescott filter

Several of the variables that we examine in the book grow over time, or are otherwise influenced by slow-moving factors. To isolate cyclical fluctuations of these variables, we first need to remove their trends. We do that using the Hodrick-Prescott (HP) filter. We pick the HP filter because it works well for our purposes, it is convenient, and it is widely used and understood in business cycle research.

The HP filter was introduced in modern macroeconomics by Hodrick and Prescott (1997), but it was discovered by mathematician Whittaker (1923), and popularized among actuaries by Henderson (1924). The Whittaker-Henderson smoothing method was well-known to macroeconomists in the 1930s (Macaulay 1931, chapter 6).

The HP filter takes a time series $y(t)$ and extracts a trend $\tau(t)$ from it, so that the initial series can be decomposed as $y(t) = \tau(t) + \delta(t)$, where $\tau(t)$ moves smoothly over time and $\delta(t)$ is the deviation of the series from trend, or in the context of business cycles, the cyclical fluctuations of the series.

Formally, given T observations of variable $y(t)$, the trend $\tau(t)$ is calculated by solving

$$\min_{\{\tau(t)\}} \left\{ \sum_{t=1}^{T} [y(t) - \tau(t)]^2 + \lambda \sum_{t=2}^{T-1} [[\tau(t+1) - \tau(t)] - [\tau(t) - \tau(t-1)]]^2 \right\}.$$

The parameter λ governs the smoothness of the trend produced by the HP filter. If $\lambda = 0$, the trend is just the series: $\tau(t) = y(t)$. When $\lambda \to \infty$, the trend is linear: $\tau(t) = g \cdot t$. In between, the trend evolves smoothly over time.

In practice, the HP filter requires that the deviation from trend, or cycle, $\delta(t)$ is a stationary process (which requires that $\delta(t)$ has a constant mean and variance). So the filter can be applied directly to a series that is not growing, such as the unemployment rate or vacancy rate. But if the series grow exponentially, such as consumption or output, then applying the HP filter directly produces cycles whose variance grows with the level of the series. That means that $\delta(t)$ is not stationary, and the decomposition no longer has the properties that the HP filter is built for. Hence, the HP filter must be applied to the logarithm of a series that is growing exponentially. Taking the logarithm of the series makes it grow linearly over time, which ensures that its cyclical component is stationary.

Before using the HP filter, we must specify a smoothing parameter λ, which determines how smooth the trend extracted by the filter is. Ravn and Uhlig (2002) recommend a smoothing parameter of 1,600 with quarterly data; this is the standard value in business cycle research. However, as Shimer (2012, p. 132) notes in the context of US labor market data, setting $\lambda = 1{,}600$ "seems to remove much of the cyclical volatility in the variable of interest." To keep more cyclicality, Shimer (2005, 2012) sets $\lambda = 100{,}000$ instead. Here, we want to keep fluctuations of interest but also apply the same filter to labor market data and to other business cycle data. So we pick an intermediate point between 1,600 and 100,000: we set the smoothing parameter to $\lambda = 10{,}000$. We use that parameter throughout the book, whenever detrending is required.

Like any filter, the HP filter has some limitations (Hamilton 2018). One limitation to keep in mind is that the HP filter struggles at the end of data samples. Accordingly, it's always good practice to compare visually the trend produced by the filter to the original, filtered series.

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