A Theory of Slack

How Economic Slack Shapes Markets, Business Cycles, and Policies

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CHAPTER 8.

Dynamics and Beveridge curve

This chapter presents a dynamic version of the slackish market model. In the dynamic model, buyers and sellers enter into long-term relationships once they have matched, as most buyers and sellers do in the real world. These relationships break down every once in a while for random reasons. To keep things simple, we focus on the equilibrium of the dynamic model. In equilibrium, market flows are balanced: there are as many new buyer-seller relationships created as destroyed at any point in time. This is a good approximation to study the day-to-day behavior of markets as long as the convergence of the market to its equilibrium is rapid—which is the case at least in the US labor market. Another advantage of focusing on the equilibrium is that a Beveridge curve appears in the model.

Before we start, a brief note about terminology: In this chapter and book I only use the word "equilibrium" to describe the point at which a variable defined through a differential equation or a collection of variables defined through a dynamical system remain invariant over time. Although this is not how the word "equilibrium" is used in modern economics, this is the standard terminology in the field of dynamical systems.[1] It is also consistent with the usage of "equilibrium" in the disequilibrium literature: the equilibrium point is where supply and demand are balanced; it is reached after an adjustment period (the disequilibrium) during which buyers and sellers learn about market conditions and prices adjust to their proper levels. Here the adjustment period occurs as market flows adjust to come into balance. The use of "equilibrium" in this book is also consistent with the

[1] See for instance the textbooks by Luenberger (1979), Perko (2001), and Hirsch, Smale, and Devaney (2013).

meaning of the word in Latin, which is roughly "state of being evenly balanced".[2]

8.1. Trading with long-term relationships

How are goods traded in this dynamic world? As we see in this section, goods are traded through long-term relationships. Because it is difficult for sellers to find buyers and buyers to find sellers, once a buyer and seller have found one another, they remain in a long-term trading relationship. Indeed, if the trading relationship is good, there is no need to look for another trading partner.

However, relationships do not last forever. At any point in time, relationships might be broken for exogenous reasons. We denote by $\lambda > 0$ the rate at which separations occur. Once there is a separation, buyers have to visit new sellers to find a new trading partner, and sellers have to display goods in their shops to attract new buyers.

8.1.1. Matching with long-term relationships

We assume that time, $t \geq 0$, is continuous. This allows us to carry out continuous-time analysis, which is simpler than discrete-time analysis.

We denote by k the number of goods for sale per unit time, and by $y(t)$ the number of goods allocated to ongoing buyer–seller relationships at time t. Each relationship delivers one good per unit time to the buyer. Since all sales occur through long-term relationships, $y(t)$ is also the number of goods sold to customers per unit time, and therefore $k - y(t)$ is the number of goods unsold and available for sale to visiting buyers per unit time.

In a dynamic world it becomes important to distinguish between visit level and visit rate, and between slack level and slack rate. The visit level is the number of visits per unit time, which we denote $V(t)$. The visit rate is the number of visits as a share of the number of goods for sale k; we denote it by $v(t) = V(t)/k$. Similarly the slack level is the number of goods unsold per unit time, which we denote $U(t) = k - y(t)$. The slack rate is the number of unsold goods as a share of the number of goods for sale, $u(t) = U(t)/k = 1 - y(t)/k$.

How does matching work for those sellers and buyers that are not in trading relationships? As usual, the number of new matches is given by a matching function, m, that takes as arguments the number of buyers' visits to sellers' shops, $V(t)$, and the number of goods for sale that are not already committed to buyers, $U(t)$. Accordingly, the number of new matches per unit time is

$$m(t) = m(V(t), U(t)).$$

[2] In economics, people would typically refer to this point as the "steady state" of the system. However, the word "steady state" has become associated in people's minds to a point that is reached in the distant future, so the terminology would be confusing here, given that the "steady state" is reached quickly.

The market tightness is as usual the ratio of the two arguments in the matching function:

(8.1) $$\theta(t) = \frac{V(t)}{U(t)}.$$

The market tightness determines the rate at which each good is sold to a new customer, $f(\theta(t))$, and the rate at which each visit leads to a purchase, $q(\theta(t))$. The rates are given by the usual expressions, (4.3) and (4.4), and have the usual properties.

8.1.2. Law of motion of trading relationships

Next, we write down the law of motion for prevailing trading relationships. The law of motion also tells us how sales vary over time in this dynamic environment, since one good is sold per unit time in each relationship. The law of motion is

$$\dot{y}(t) = f(\theta(t))\,[k - y(t)] - \lambda y(t).$$

In words, the change in the number of relationships ($\dot{y}(t)$) is equal to the number of new customer relationships ($f(\theta(t))\,[k - y(t)]$) minus the number of existing customer relationships that broke down ($\lambda y(t)$).

From these dynamics, we infer the law of motion for the rate of slack, $u(t)$. The rate of slack is the share of goods that are available for sale but unsold:

(8.2) $$u(t) = \frac{k - y(t)}{k} = 1 - \frac{y(t)}{k}.$$

Accordingly, the law of motion of the slack rate follows from the law of motion for the number of trading relationships:

$$\dot{u}(t) = -\frac{\dot{y}(t)}{k} = \lambda\frac{y(t)}{k} - f(\theta(t))(1 - \frac{y(t)}{k}).$$

This gives the law of motion for the slack rate:

(8.3) $$\dot{u}(t) = \lambda\,[1 - u(t)] - f(\theta(t))u(t).$$

The law of motion says that the slack rate increases when trading relationships are severed ($\lambda\,[1 - u(t)]$) and decreases when new trading relationships are formed ($f(\theta(t))u(t)$).

8.2. Beveridge curve

Now that we have the differential equation describing the dynamics of the slack rate, we can solve it to determine what the slack rate converges to, and how quickly it does so. This is what we do in this section. We find that the Beveridge curve—the negative relationship between visits and slack—naturally arises in a dynamic slackish model, as the equilibrium point of the law of motion of slack.

8.2.1. Building a Beveridge curve from balanced flows

For the analysis here, we consider that market tightness θ and thus the selling rate $f(\theta)$ are constant. Given those, we study the dynamics of the slack rate.

First, let us determine the equilibrium point of the differential equation (8.3). By definition, the equilibrium is the slack rate u such that $\dot{u} = 0$. Thus, the equilibrium slack rate satisfies

$$-\left[\lambda + f(\theta)\right] u + \lambda = 0,$$

which implies that the equilibrium slack rate is solely determined by the selling rate $f(\theta)$ and separation rate λ:

$$u = \frac{\lambda}{\lambda + f(\theta)}. \tag{8.4}$$

In fact, the equilibrium point is decreasing in the selling rate and therefore market tightness. The intuition of course is that when goods are sold more rapidly, fewer goods remain unsold at any point in time, so there is less slack. The equilibrium point is increasing in the separation rate, on the other hand. Here the intuition is that when trading relationships break down more often, more goods are unattached and therefore unsold at any point in time, so there is more slack.[3]

Furthermore, the relationship between slack rate and tightness given by (8.4) is a Beveridge curve: it describes a negative relationship between slack rate and visit rate. This can be seen by introducing visits into the equation. Visits are implicitly present because market tightness is just the ratio of visit rate to slack rate: $\theta = v/u$. We start by rewriting our equation (8.4):

$$\lambda u + f(\theta) u = \lambda.$$

The selling rate is given by $f(\theta) = m(1, \theta)$, as we saw with equation (4.3). Since the matching function has constant returns to scale, $f(\theta) u = m(u, \theta u) = m(u, v)$. Hence, the equilibrium condition for the slack rate links the visit and slack rates by

$$m(u, v) = \lambda(1 - u). \tag{8.5}$$

The equilibrium condition is called a balanced-flow condition and we see clearly why here: the slack rate is in equilibrium when the number of new trading relationships, given by

[3] Recall here that the function $x/(x + f)$ is increasing in $x > 0$.

the matching function $m(u, v)$, equals the number of trading relationships that separate, which itself is given by the share of goods in relationships, $1 - u$, times the rate at which those separate, λ. So, slack is constant over time when the numbers of relationships that form and separate are equal.

How can we see that the balanced-flow equation (8.5) is a Beveridge curve? Well, it suffices to differentiate it implicitly. The equation implicitly defines the visit rate as a function $v(u)$ of the slack rate. Implicitly differentiating the equation gives

$$\frac{\partial m}{\partial u} + \frac{\partial m}{\partial v} \cdot v'(u) = -\lambda. \tag{8.6}$$

Reshuffling terms to isolate the derivative $v'(u)$, we get:

$$v'(u) = -\frac{\lambda + \partial m/\partial u}{\partial m/\partial v}. \tag{8.7}$$

Since the matching function is increasing in both arguments, we have $\partial m/\partial u > 0$ and $\partial m/\partial v > 0$, which tells us that $v'(u) < 0$. The balanced-flow equation therefore generates a Beveridge curve: it defines the visit rate as a decreasing function of the slack rate. The main reason is that when there is more slack, more goods are available to be sold, so given the properties of the matching function, fewer visits are necessary to generate the required number of new matches. A secondary reason is that when there is more slack, there are fewer established trading relationships, so fewer separations for a given separation rate. Fewer new matches are then required to compensate these separations.

8.2.2. Convergence to the Beveridge curve

We have now seen that the Beveridge curve appears at the point where the market flows—creation and destruction of trading relationships—are balanced. The next question is how long it takes for the market to converge to the Beveridge curve; that is, how long it takes for the slack rate $u(t)$ to converge to the equilibrium point $u(\theta)$, defined by (8.4).

We can rewrite the differential equation as

$$\dot{u}(t) + (\lambda + f)u(t) = \lambda.$$

This is a first-order linear differential equation, which is easily solvable. The solution takes the form:

$$[u(t) - u(\theta)] = [u(0) - u(\theta)]\, e^{-(\lambda+f)t}.$$

It is easy to verify that this solution satisfies the differential equation and the initial condition at $t = 0$. The interpretation of the solution is that the gap between the slack rate $u(t)$ and the equilibrium point $u(\theta)$ shrinks at a rate of $\lambda + f$. So if the selling rate f

and separation rate λ are large enough, convergence to the equilibrium—to the Beveridge curve—is rapid.

The speed of the convergence to the Beveridge curve depends on the market that we consider. In some markets selling is faster or there is more turnover in relationships, so convergence is faster. In other markets, selling takes more time or relationships are stabler, so convergence is slower.

In this book, we take the perspective that convergence to the Beveridge curve is fast enough that we can ignore it. The motivation for the assumption comes from the US labor market. There, λ is roughly 3% per month, f is around 56% per month, so $\lambda + f$ is about 59% per month (Michaillat and Saez 2021, p. 7). This means that the gap between the unemployment rate and the Beveridge curve shrinks very quickly. The half-life of this exponential decay is the time it takes to halve the initial distance between the unemployment rate and the Beveridge curve. It is given by the following expression:

$$\frac{\ln(2)}{\lambda + f} = \frac{0.69}{0.59} \approx 1.2.$$

This means that it will take only 1.2 months to halve the initial distance between the unemployment rate and the Beveridge curve. This is an extremely fast convergence to the Beveridge curve—after one quarter, only $(1/2)^3 = 1/8$ of the initial distance will be left.

We also make the assumption because it greatly simplifies the analysis of the dynamic model, since it replaces a differential equation (equation (8.3)) by a static relationship (equation (8.4)). Thus, we can simplify our model by getting rid of the dynamics of slack.[4]

The assumption that at the time scale of the model market flows are always balanced simply neglects the short period of time during which flows are unbalanced, before the market converges to its balanced state. This is very much like the assumption that people are always rational in economic models—which neglects the learning period before people converge to a rational behavior. It is also akin to the assumption that people always play the Nash equilibrium in game theory—which neglects the learning or coordination period that it takes to reach such equilibrium.

In any dynamic model, we therefore assume that market flows are balanced at all times so the slack rate is just a function of the market tightness:

$$u(\theta) = \frac{\lambda}{\lambda + f(\theta)}. \tag{8.8}$$

The qualitative properties of the equilibrium slack rate are the same as the properties of the slack rate in the static model: when market tightness is 0, the slack rate is 1; then the

[4]Just as we do here, it is common in the DMP literature to assume that the Beveridge curve holds at all times. See for instance Pissarides (1986), Pissarides (2009), Hall (2005b), Hall (2005a), and Elsby, Michaels, and Solon (2009).

slack rate is a decreasing and convex function of tightness. These properties follow from the properties of the selling rate, $f(\theta)$, which is 0 when tightness is 0 and is increasing and concave in tightness. To establish convexity, we also use the facts that the function $\lambda/(\lambda+x)$ is decreasing and convex in $x \geq 0$, that the composite of a function that is convex and decreasing with a concave function is convex.

Figure 8.1A displays the equilibrium slack rate as a function of tightness to illustrate its properties in this dynamic model.

8.2.3. Why can slack never disappear?

The expression for the slack rate (8.8) explains why there is always some slack in the market: in a dynamic world, slack can never completely disappear. Mathematically, the reason is that since the separation rate λ is positive and the selling rate $f(\theta)$ is finite, the equilibrium slack rate $u(\theta)$ is bound to be positive.

Intuitively, the reason is that at any moment some trading relationships are bound to be terminated—because the circumstances of some buyers and sellers constantly change so that they must quit their relationships. The goods that were traded in these relationships become available again, but it takes some time for sellers to find new customers. As a result, the goods remain unsold for some time: there is some slack on the market.

In the context of the labor market, it has been known for a long time that labor-market flows impose a minimum level of unemployment: unemployment can never completely disappear. For instance, Robinson (1946, pp. 169–170) made the following observation:

> In a changing world there are always bound to be, at any moment, some workers who have left one job and have not yet found another.... Changes in occupation for personal reasons will always be going on. So long as such shifts in employment are taking place there is always likely to be some unemployment even when the general demand for labor is very high.

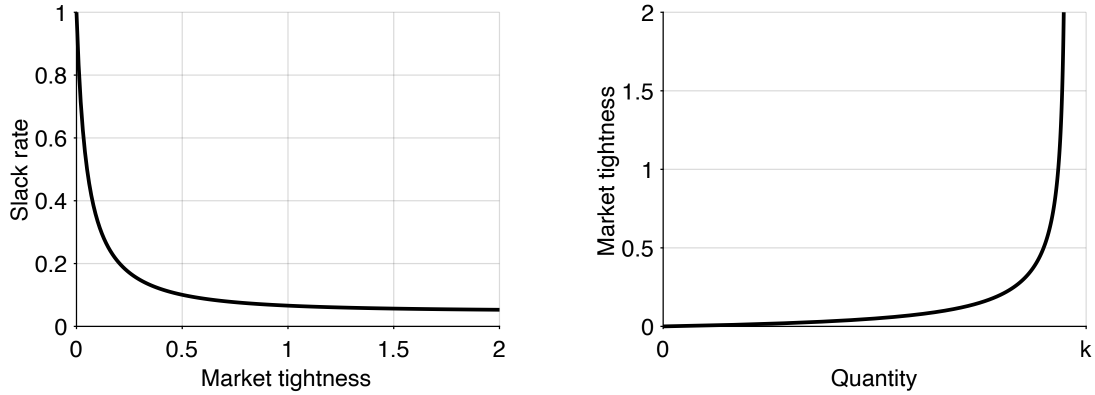
8.3. Equilibrium market supply

Now that we have computed the equilibrium slack rate, it is easy to compute the market supply. Indeed, output $y(t)$, slack rate $u(t)$, and market capacity k are related by

$$y(t) = [1 - u(t)]\, k,$$

because by definition the slack rate is the share of capacity that is unsold at any time (which is also formalized by (8.2)).

But in equilibrium, the slack rate is related to tightness by (8.8), so that output is a function of tightness too. This function of tightness gives the amount of output sold



A. Slack rate

B. Market supply

FIGURE 8.1. Numerical illustration of the dynamic slackish market model in equilibrium

The slack rate is given by (8.8). The market supply is given by (8.9). The matching function is CES, given by (4.10). Parameters are set to $\sigma = 2$, $\kappa = 4$, $k = 1$, $\alpha = 0.5$, $a = 2$, $p = 1$, and $\lambda = 5\%$.

through the matching process when flows are balanced. It is the equilibrium market supply in this dynamic world, and it is given by

$$y^s(\theta) = [1 - u(\theta)]\, k, \tag{8.9}$$

where $u(\theta)$ is the equilibrium slack rate given by (8.8), and k is the market capacity.

The qualitative properties of the equilibrium market supply are the same as the properties in the static model: when market tightness is 0, the supply is 0; then the supply is increasing and concave in tightness. These properties follow from the properties of the equilibrium slack rate, which is 1 when tightness is 0 and is decreasing and convex in tightness.

We plot again the market supply curve in a tightness-quantity plane to visualize its behavior in this dynamic model (figure 8.1B). Compared to the market supply curve in a static model (figure 5.1D), this market supply curve is much more curved—although both supply curves are obtained with the same CES matching function, under the same calibration.

We can see why the market supply curve is more curved in the dynamic world by comparing the expressions for the two supply curves. Once the expressions (5.4) and (8.8) for the slack rates are substituted into the market supplies, the supplies in the static and dynamic models are given by

$$y^s(\theta) = f(\theta)k \quad \text{and} \quad y^s(\theta) = \frac{f(\theta)}{\lambda + f(\theta}k.$$

Hence, in a dynamic model, the concave function $f(\theta)$ is composed with a concave and increasing function, $x/(x+\lambda)$, which makes the market supply $y^s(\theta)$ a more concave function of tightness.

Because of this sharp curvature of the market supply, the slackish model is sharply state dependent. This state dependence has a range of implications, because it implies that the model behaves very differently when tightness is high and when tightness is low. Section 8.8 describes the first manifestations of this state dependence by contrasting the effects of supply and demand shocks in tight and slack markets.

8.4. Equilibrium matching wedge

Now switching to the demand side of the model, our first step is to compute the matching wedge: the share of consumption that must be devoted to matching on a slackish market. In the dynamic model, the matching wedge has a slightly different expression. Here we maintain the assumption that the market is in equilibrium, so the market remains the same at any point in time. We therefore omit the time index to keep notation simpler.

Consider a buyer j who conducts V_j visits to consume c_j goods. Because the buyer must also purchase some goods for matching, their purchases are larger than their consumption: $y_j > c_j$. Since we assume that market flows are balanced, the y_j trading relationships through which the buyer purchases the y_j goods remains stable over time, which requires that the number of new relationships created at any point in time must be equal to the number of trading relationships that are destroyed at that point. The number of relationships that are destroyed is λy_j, as λ is the separation rate, while the number of relationships that are created is $q(\theta)V_j$, as $q(\theta)$ is the buying rate. Thus, to maintain y_j relationships with sellers, a buyer must visit

$$V_j = \frac{\lambda y_j}{q(\theta)} \tag{8.10}$$

shops. Given that each visit requires κ goods, output and consumption are related by

$$y_j = c_j + \frac{\kappa\lambda y_j}{q(\theta)}. \tag{8.11}$$

The equation relating purchases and consumption in this dynamic world (equation (8.11)) is the same as the equation in the static world (equation (5.6)), except that the term κ in the static world becomes $\kappa\lambda$ in the dynamic world. Therefore, the matching wedge relating purchases to consumption admits the same expression as in the static model once κ is replaced by $\kappa\lambda$. Thus, $y_j = [1+\tau(\theta)]c_j$ where

$$\tau(\theta) = \frac{\kappa\lambda}{q(\theta) - \kappa\lambda}. \tag{8.12}$$

The matching wedge has the same properties as in the static model except that κ must be replaced by $\kappa\lambda$. For instance, the upper bound $\bar{\theta}$ on tightness, at which $\tau(\theta) \to \infty$, is defined by $\bar{\theta} = q^{-1}(\kappa\lambda)$ instead of $\bar{\theta} = q^{-1}(\kappa)$ in the static model.

8.5. Equilibrium market demand

In a dynamic world, buyers ought to have a utility function that integrates consumption at all times, discounting future consumption appropriately. Then each buyer would maximize this present-discounted utility subject to their intertemporal budget constraint, which allows buyers to use money to smooth consumption over time. However, the dynamics along the convergent path to the equilibrium, where the buyer's consumption is constant over time, are not relevant to the present market model. To abstract from them, we simply assume that agents' time discount rate equals 0. This simplifies the analysis by allowing us to directly focus on the equilibrium of the buyer's problem, rather than having to solve an optimal control problem.[5]

Under the assumption of zero discount rate, the buyer's problem is just a static problem: the buyer simply maximizes the instantaneous utility subject to the equilibrium budget constraint, without any intertemporal considerations. Once we move to a model of the entire economy (part III), we will of course allow for saving and intertemporal dynamics—but these are not the object of the present analysis.

Accordingly, we simply assume that each buyer maximizes utility (5.10) subject to the budget constraint (5.11), where the matching wedge is given by (8.12). This is the same problem as in the static model, so the market demand remains the same, given by (5.15), with the same properties.

8.6. Solution of the model

Now that we have introduced all the elements of the dynamic model, we are in a position to solve it. The structure of the model remains the same as in the basic model, but with a few adjustments, as described in table 8.1.

The approach to find the solution of the dynamic model is the same as in the static case: first, determine the market tightness, θ; then, compute the values of the 5 remaining variables (c, p, y, u, v).

[5] Pissarides (1984) and Hosios (1990) make the same assumption in their analysis of labor market efficiency: they set discount rate to 0 to focus on the equilibrium of the model and abstract from transition dynamics. Another equivalent assumption would be that all money not spent (wealth) is taxed at the end of each period by the government, and transfers of money are made at the beginning of each period. Then buyers would only face static—not intertemporal—constraints so they would simply maximize their flow utility at any point in time, irrespective of their discount rates.

TABLE 8.1. Structure of the dynamic slackish market model in equilibrium

Variable	Name	Condition	Interpretation	Reference
c	Consumption of goods	$c = c^d(\theta, p)$	Utility maximization	(5.14)
p	Price of goods	$p = p^n(\theta)$	Price norm	(5.16)
y	Sales of goods	$y = [1 + \tau(\theta)]c$	Matching wedge	(5.7)
v	Visit rate	$v = \lambda y/[q(\theta)k]$	Balanced flows	(8.10)
θ	Market tightness	$\theta = v/u$	Definition of tightness	(8.1)
u	Slack rate	$u = \lambda/[f(\theta) + \lambda]$	Beveridge curve	(8.8)

Just as in the static model, tightness is determined from a supply-equals-demand condition in the dynamic model. Indeed, the amount of output demanded by buyers imposes that $y = y^d(\theta, p^n(\theta))$, where the equilibrium market demand is given by (5.15). At the same time, output is related to tightness via the equilibrium market supply, $y = y^s(\theta)$, where the equilibrium market supply is given by (8.9). The market tightness that solves the model equalizes equilibrium market demand and supply: $y^d(\theta, p^n(\theta)) = y^s(\theta)$. The main difference here with the static model is that the market supply is substantially different. Nevertheless, graphically, the solution of the model is found at the intersection of the market supply and demand curves. The equilibrium market supply curve captures the number of trades, for any tightness, given market capacity and balanced flows. The equilibrium market demand curve captures the number of trades, for any tightness, that maximize the per-period utility of buyers given their budget constraints.

8.7. Comparative statics

In this section, we solve the model under rigid prices (which includes the special case of fixed prices) and analyze the response of the model to demand and supply shocks. The analysis will be brief as most of the results are the same as in the static model. We will only make small adjustments to account for the shape of the market supply in this dynamic model.

In this dynamic world the rigid prices remain given by (6.1), and the fixed price is a special case of the rigid price with $\gamma = 1$. The expression for the rigid price does not need to be amended in this dynamic model because the structures of the market demand and supply are close enough to those in the static model. In fact, the rigid price only enters the market demand, which remains given by (6.2) in this dynamic model.

The model behavior is determined by the supply-equals-demand condition, which pins down market tightness. Collecting all our results so far, and fetching results from the static model that continue to apply, we see that this condition takes the following form here:

$$\left[\frac{(1-\alpha)a^{\gamma}}{\rho}\right]^{1/\alpha}\frac{k^{1-\gamma}}{[1+\tau(\theta)]^{1/\alpha-1}}=\frac{f(\theta)}{\lambda+f(\theta)}\cdot k.$$

The left-hand side is the market demand, which comes from (6.2), and the right-hand side is the market supply, which comes from (8.9).

Formally, this condition is the same as condition (6.3) in the static case, with two changes. First, in the matching wedge $\tau(\theta)$, the term κ is replaced by the term $\kappa\lambda$, which incorporates the separation of relationships. This change does not modify any of the properties of the model, since none of the properties of the model rely on the specific value of the matching cost κ. Second, in the market supply, the term $f(\theta)$ is replaced by $f(\theta)/[\lambda+f(\theta)]$, which reflects the flows on the market. This change does not affect the qualitative properties of the model, because both functions f and $f/(\lambda+f)$ have the same qualitative properties: they are 0 when $\theta=0$ and are increasing and concave in θ. Quantitatively, however, the function $f/(\lambda+f)$ is much more curved than f, which will influence how shocks affect the market in good and bad times.

We can quickly run through the properties of the dynamic model, using our analysis of the static model under fixed price in chapter 5 and under rigid prices in chapter 6. First, we see that the model admits a unique solution, through the same logic as in the static model with a fixed price, which is illustrated in figure 5.3. After a negative demand shock (a reduction in the demand parameter a), the market demand curve moves inward while the market supply curve remains unmoved, as shown in figure 6.1A, so tightness and sales fall. After a negative supply shock (a reduction in the market capacity k), the market supply shifts inward. Because prices increase a little when goods are scarcer—except if they are completely fixed—the market demand is depressed too, as illustrated in figure 6.1B. Overall, however, whether prices are fixed or only rigid, tightness increases and sales decreases.

Despite the formal similarities between the static and dynamic models, the market adjusts to shocks slightly differently, and maybe not entirely as expected, in the dynamic world. Consider for instance a negative demand shock: suddenly buyers do not value the goods offered on the market as much. We have seen that market tightness drops so the slack rate rises: more goods remain unsold. One might imagine that more goods are unsold because more buyers terminate their relationships with sellers. But in the model this is not what happens: the reason why more goods remain unsold is that buyers reduce the number of new relationships they form. It becomes harder for sellers to find buyers for their goods, so the slack rate goes up. Existing buyer-seller relationships continue to operate as before, but their number dwindles by natural attrition. And because fewer new relationships are created at any point in time, there is more slack on the market: the stock of unsold goods balloons.

8.8. State dependence

One aspect of the model that is exacerbated in the dynamic environment is the curvature of the market supply curve. The market supply curve is much more curved—much more nonlinear—in the dynamic model, as can be seen by comparing figures 5.1D and 8.1B. This mathematical property—a highly curved market supply—has important economic implications. It means that the model behaves very differently when the market is slack or tight: the model is very state dependent. In this section we formalize and explain this state dependence. Many more implications will appear when we discuss stabilization policies in part IV.

8.8.1. Curvature of the market supply

The entire approach followed in chapter 6 to compute the response of tightness and other variables to demand and supply shocks remains valid. The one key difference is that the market supply has a different curvature, so the elasticity of market supply with respect to tightness is different, which will modify the results. Using the results on elasticities from appendix B, the fact that the elasticity of $f(\theta)$ with respect to θ is $1-\eta$, as well as the expression (8.9) for the market supply, we obtain the following elasticity of market supply with respect to tightness:

$$\epsilon^s_\theta = [1-\eta(\theta)] - \frac{f(\theta)}{f(\theta)+\lambda}[1-\eta(\theta)] = [1-\eta(\theta)]\frac{\lambda}{f(\theta)+\lambda} = [1-\eta(\theta)]u(\theta). \tag{8.13}$$

The market supply is more curved in this dynamic environment because of the term $u(\theta)$ in the elasticity. When the slack rate is 1, the elasticity is just $1-h$, as in the static case. But as the slack rate falls, the elasticity drops, which means that the supply is less and less responsive to tightness. The elasticity converges to 0 when the slack rate goes to 0: at that point the market supply is perfectly vertical in a tightness-quantity plane.

8.8.2. Response to demand shocks

The sharp state dependence of the dynamic slackish model is clearly visible once we look at the response of sales to shocks. Let us start by looking at the response of sales to a demand shock. The response is given by the elasticity $\epsilon^y_a = \epsilon^s_\theta \cdot \epsilon^\theta_a$ where ϵ^θ_a is given by (6.9). This gives

$$\epsilon^y_a = \frac{\epsilon^d_a}{1-\epsilon^d_\theta/\epsilon^s_\theta}$$

We plug into these equations the appropriate values for the elasticities, which are provided in chapter 6 except for the elasticity ϵ^s_θ which is given by (8.13). We get

$$\epsilon^y_a = \frac{\gamma}{\alpha + (1-\alpha)\cdot\frac{\eta(\theta)}{1-\eta(\theta)}\cdot\frac{\tau(\theta)}{u(\theta)}}. \tag{8.14}$$

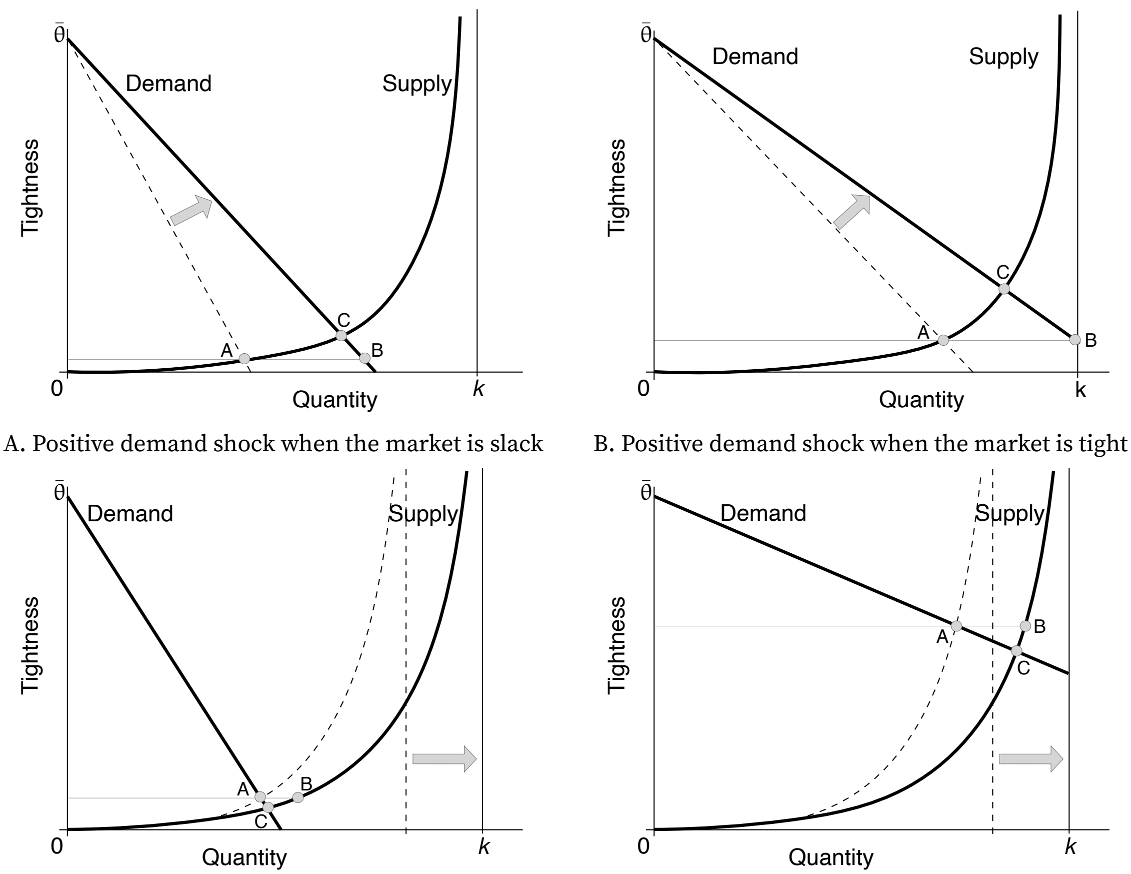
We see that demand shocks have larger effects on output when the market is slack. The matching wedge $\tau(\theta)$ is increasing in tightness, and the slack rate $u(\theta)$ is decreasing in tightness, so the ratio $\tau(\theta)/u(\theta)$ is increasing in tightness. Furthermore, as we saw in chapter 6, for the matching functions that we have considered in chapter 4, the ratio $\eta(\theta)/[1-\eta(\theta)]$ is weakly increasing in tightness. Hence, the product of the ratios $\eta(\theta)/[1-\eta(\theta)]$ and $\tau(\theta)/u(\theta)$ is strictly increasing in tightness. This implies that, since all other terms in the elasticity are constant, the elasticity ϵ^y_a is decreasing in tightness. This means that when the market is slack (low tightness), the elasticity is high: output responds sharply to changes in market demand. By contrast, when the market is tight (high tightness), the elasticity is low: output does not respond much to changes in market demand. Furthermore, the cyclicality of the elasticity ϵ^y_a is exacerbated in the dynamic world because of the presence of the countercyclical unemployment rate $u(\theta)$ in (8.14). The term $u(\theta)$ captures the extra curvature of the market supply in a dynamic world.

The result that demand shocks have larger effects when the market is slack is intuitive. Consider an increase in demand. If tightness remained the same, that increase in demand would directly translate into an increase in output. In figures 8.2A and 8.2B, that increase corresponds to the distance A–B. However, to generate the additional matches required to trade the additional output, more visits are required. These additional visits raise market tightness, which make consumption less attractive—as there is more competition among buyers for the goods for sale. The increase in tightness therefore somewhat reduces output demanded: the reduction corresponds to the movement B–C along the market demand.

In a slack market, depicted in figure 8.2A, visits are highly successful, so few extra visits are required to generate the additional trades. Hence, tightness does not increase much, so the reduction B–C is small, and the overall increase in output, A–C, is large. This means that demand disturbances have large effects on output in slack markets.

In a tight market, depicted in figure 8.2B, things are different. Then, visits are much less likely to be successful, so many more visits are required to generate the additional trades. Tightness increases significantly, so the reduction B–C is large, which means that the overall increase in output, A–C, is small. The implication is that demand disturbances have small effects on output in tight markets.

To illustrate the state dependence of the dynamic slackish model, we plot the demand elasticity of output, ϵ^y_a (figure 8.3A). We see that when tightness increases from 0 to 2, the demand elasticity of output drops sharply, from 2 to 0. The numerical values of the



A. Positive demand shock when the market is slack

B. Positive demand shock when the market is tight

C. Positive supply shock when the market is slack

D. Positive supply shock when the market is tight

FIGURE 8.2. State dependence in the dynamic slackish market model

The market supply is given by (8.9). The market demand is given by (6.2). The price norm is given by (6.1). A negative demand shock is a reduction in the preference for goods a. A negative supply shock is a reduction in market capacity k.

elasticity would depend on the calibration of the model, and would change for instance if the price rigidity γ was calibrated differently. The goal of the numerical exercise is only to illustrate that the slackish model's state dependence is strikingly strong.

8.8.3. Response to supply shocks

We obtain opposite results with supply shocks: supply shocks have larger effects on output when the market is tight.

The response of output to supply shocks is given by the elasticity $\epsilon^y_k = \epsilon^s_k + \epsilon^s_\theta \cdot \epsilon^\theta_k$ where ϵ^θ_k is given by (6.11). This gives

$$\epsilon^y_k = \epsilon^s_k + \frac{\epsilon^d_k - \epsilon^s_k}{1 - \epsilon^d_\theta / \epsilon^s_\theta}$$

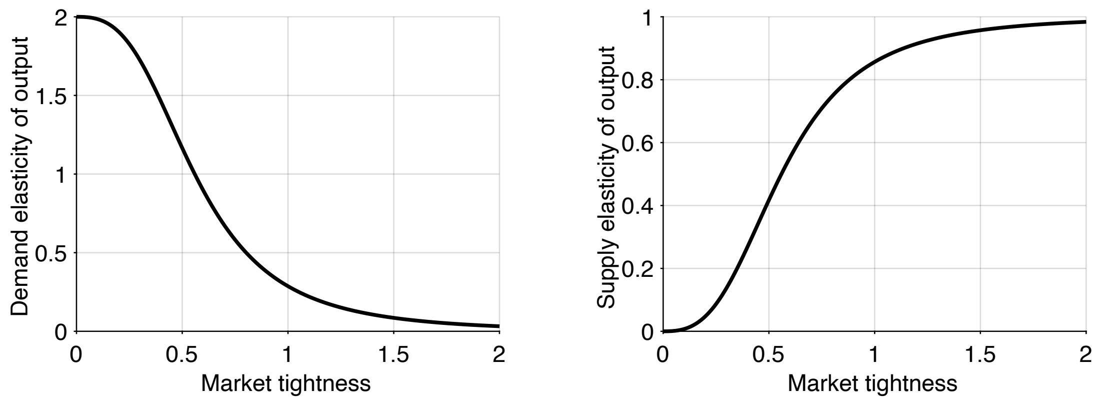
We plug into these equations the appropriate values for the elasticities, which are provided in chapter 6, except for the elasticity ϵ^s_θ, which is given by (8.13). We get

$$\epsilon^y_k = 1 - \frac{\gamma}{1 + \frac{1-\alpha}{\alpha} \cdot \frac{\eta(\theta)}{1-\eta(\theta)} \cdot \frac{\tau(\theta)}{u(\theta)}}. \tag{8.15}$$

Following the same logic as above, we see that the response of output to supply shocks is stronger in a tighter market. We know that the product of the terms $\eta(\theta)/[1-\eta(\theta)]$ and $\tau(\theta)/u(\theta)$ is strictly increasing in tightness. Hence, the elasticity ϵ^y_k is increasing in tightness. This means that when the market is tight, the elasticity is high: output responds significantly to changes in market capacity. By contrast, when the market is slack, the elasticity is low: output does not respond much to changes in market capacity.

To understand the result, consider an increase in market capacity. If tightness remained the same, that increase in supply would directly translate into an increase in output. However, because the market demand has not changed, buyers would not be willing to absorb this output at the current tightness. In figures 8.2C and 8.2D, that gap between demand and supply corresponds to the distance A–B. Buyers thus start reducing their visits to reduce their purchases, which lowers tightness. This reduction in tightness has two implications: Buyers are willing to purchase more goods than before (a movement down the demand curve from A), and fewer matches are made so fewer goods are traded (a movement down the supply curve from B). Overall, the gap between demand and supply shrinks until the market reaches C, where demand and supply are equalized again. The overall increase in output from the increase in supply is the distance A–C, which is less than A–B.

In a tight market, depicted in figure 8.2D, the supply curve is relatively steep and the demand curve is relatively flat, so the new output level is much closer to point B than



A. Elasticity of output to demand

B. Elasticity of output to supply

FIGURE 8.3. Numerical illustration of state dependence in the dynamic slackish market model

The elasticity of output to demand is given by (8.14). The elasticity of output to supply is given by (8.15). The matching function is CES, given by (4.10). Parameters are set to $\sigma = 2$, $\kappa = 4$, $k = 1$, $\alpha = 0.5$, $a = 2$, $p = 1$, $\gamma = 1$, and $\lambda = 5\%$.

point A. This means that the increase in output, A–C, is large: supply disturbances have large effects on output in tight markets.

In a slack market, depicted in figure 8.2C, things are different. The supply curve is relatively flat and the demand curve is relatively steep, so the new output level is much closer to point A than point B. This means that the increase in output, A–C, is small: supply disturbances have small effects on output in slack markets.

To illustrate further the state dependence of the dynamic slackish model, we plot the supply elasticity of output, ϵ^y_k (figure 8.3B). We see that when tightness increases from 0 to 2, the supply elasticity of output rises sharply, from 0 to 2. Once again, the numerical exercise illustrates that the slackish model is strongly state dependent.

8.9. Summary

This chapter presents a dynamic version of the slackish market model. Overall, the structure of the dynamic slackish model is almost identical to that of the static slackish model. The main difference is that there are flows on the market—new customer relationships being created and destroyed. In equilibrium, however, these flows are balanced, and the model has a very similar structure to the static model. The equilibrium of the model can be analyzed with supply-and-demand diagrams, just like the static model.

The Beveridge curve emerges naturally in the dynamic model at the equilibrium: it links visits to slack when the number of new trading relationships formed equals the number of relationships that separate. Empirical evidence from the US labor market suggests convergence to the Beveridge curve is rapid, justifying the assumption that markets operate in equilibrium. This assumption considerably simplifies the analysis by replacing differential equations with static relationships.

In a dynamic world, the market supply is a more curved function of tightness than in the static model, which generates strong state dependence in the model's responses to shocks. Demand shocks have larger effects when the market is slack, while supply shocks have large effects when the market is tight. This state dependence implies that the same disturbance has very different consequences depending on the prevailing state of the market, which will become important in later chapters when analyzing stabilization policy.

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