A Theory of Slack

How Economic Slack Shapes Markets, Business Cycles, and Policies

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CHAPTER 7.

Endogenous capacity

So far we have assumed that the market capacity—number of goods sold on the market—is given exogenously. The motivation is that supply decisions do not adjust much in the short run. It takes time to systematically supply more or fewer goods.

Nevertheless, to understand completely how slackish markets work, it is good to examine how the model works when the capacity is endogenous. This is what we do in this chapter. The case with endogenous capacity will be helpful in several situations later in the book, for instance when we study the efficient rate of slack (chapter 9).

7.1. Market supply with endogenous capacity

The cornerstone of the supply side is the seller's optimization problem: choosing how much to sell so as to maximize utility. We set up and solve the seller's problem, and then derive the properties of the market supply.

7.1.1. Seller's utility function

Once again there is a mass 1 of sellers. Each seller $i \in [0,1]$ can either enter the market to sell k goods, or remain outside of the market and not sell anything. The seller must decide before the market opens to enter or not. If they enter, they place k goods for sale, and sell a fraction of them. If they do not enter, they do not place anything for sale, but they enjoy utility ξi^{ϕ}. The parameter $\xi > 0$ governs the utility from nonparticipation relative to money. The parameter $\phi > 0$ ensures that the utility from nonparticipation is increasing in i. Sellers with high i enjoy nonparticipation—leisure or recreation—very much. Sellers with low i enjoy working instead of leisure. In addition all sellers have linear utility over money balances.

A seller's only decision is whether to enter the market or not. The number of sellers who enter the market is $h \in [0, 1]$. If person i refuses to enter, she receives no income but enjoys leisure utility ξi^{ϕ}. If a seller decides to enter, she receives utility from her expected income but no utility from leisure. A fraction $f(\theta)$ of all goods are sold at price p, where $f(\theta)$ is the selling probability. So any seller's expected income is simply $f(\theta)pk$.

7.1.2. Market tightness

Importantly, because the number of goods for sale is $\int_0^h k\,di = hk$, the matching function says that the number of trades on this market is

$$y = m(v, hk).$$

Hence, the market tightness in the case with endogenous capacity is defined as

$$\theta = \frac{v}{hk}. \tag{7.1}$$

7.1.3. Seller's problem

The seller's problem is simple: the seller must simply decide to participate or not. The h sellers with sufficiently low utility from nonparticipation enter the market. The remaining $1-h$ sellers, who have sufficiently high utility from nonparticipation relative to the expected market income, abstain from entering.

Formally, people opt to participate when $\xi i^{\phi} \leq f(\theta)pk$, and they decline to participate when $\xi i^{\phi} > f(\theta)pk$. Accordingly, the capacity of the market is implicitly defined by

$$\xi h^{\phi} = f(\theta)pk.$$

This equation says that the marginal market participant ($i = h$) is indifferent between participating and not, because her nonparticipation utility (ξh^{ϕ}) equals the expected income ($f(\theta)pk$). Reshuffling the equation, we express the market participation rate as a function of market tightness and market price:

$$h = \left[\frac{f(\theta)pk}{\xi}\right]^{1/\phi}. \tag{7.2}$$

The participation rate is increasing in market tightness and market price because both raise sellers' expected income.

To illustrate, we plot the participation rate h as a function of tightness (figure 7.1A). To ensure that $h \leq 1$ for all θ, we assume that the parameter ξ is large enough. We want $h \leq 1$ even when $f(\theta) = 1$, which requires $\xi \geq pk$.

Before proceeding further, how can we reconcile the finding in chapter 4 that the US labor market tightness is procyclical, with the evidence presented in chapter 3 that the US labor force participation rate is roughly acyclical? According to equation (7.2), we would expect the US labor force participation rate to be procyclical, since $f(\theta)$ is procyclical? This is correct, but we would expect the fluctuations to be small because at the extensive margin, labor supply is quite inelastic, which means that the Frisch elasticity of labor supply $1/\phi$ is quite small (Chetty et al. 2012). Through (7.2), this inelasticity implies that the participation rate does not respond much to labor market tightness. This might explain why, even if participation is endogenous, the fluctuations in labor force participation rate are hard to discern.

7.1.4. Computing the market supply

On the market, we have h sellers, each selling k goods. The notional supply kh is not fixed any more: it depends on the market price and tightness via equation (7.2).

Once again, we use the effective market supply, which is the amount of goods actually sold given tightness, and which is less than the notional supply because sellers are unable to sell all the goods that they put for sale. The effective supply is $f(\theta)kh < kh$. From equation (7.2), it is pretty simple to express the market supply:

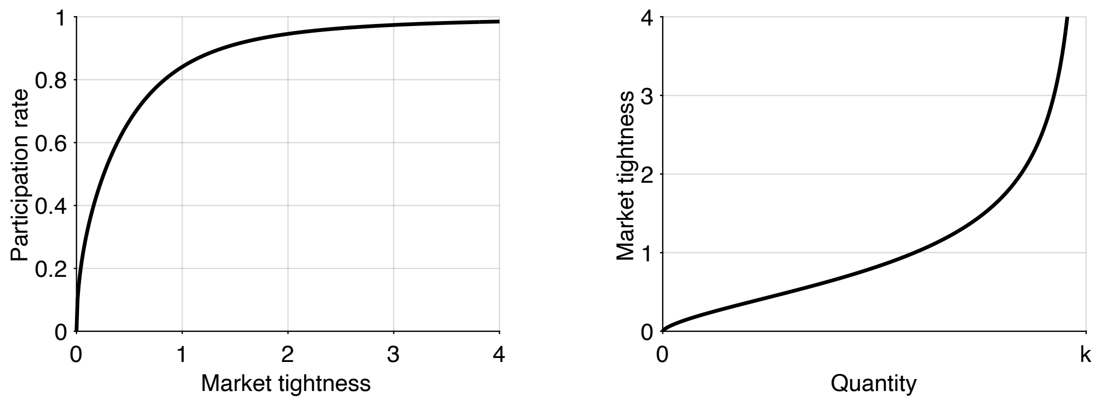
$$y^s(\theta, p) = [f(\theta)k]^{1+1/\phi} \left(\frac{p}{\xi}\right)^{1/\phi}. \tag{7.3}$$

Notice that when $\phi \to \infty$, the market supply with endogenous capacity reduces to the market supply with fixed capacity, $y^s(\theta) = f(\theta)k$. This is because in this case, the utility of leisure ξi^{ϕ} goes to 0 for all sellers $i < 1$, so all sellers enter the market ($h = 1$), and the market capacity is fixed at k.

7.1.5. Properties of the market supply

Let us now look at the properties of the market supply, $y^s(\theta, p)$. First, we see that it is increasing in the market price p. When the price is higher, entering the market becomes more attractive, so supply is higher.

We also see that when market tightness θ is 0, the market supply is 0, because $f(0) = 0$. Then, the market supply is increasing in market tightness. This is visible in (7.3) because the selling probability $f(\theta)$ is increasing in tightness. This is because when the market is tighter, more goods are sold, which has a mechanical effect on supply, and also provides



A. Participation rate

B. Market supply

FIGURE 7.1. Numerical illustration of the slackish market model with endogenous capacity

The market participation rate is given by (7.2). The market supply is given by (7.3). The matching function is CES, given by (4.10). Parameters are set to $\gamma = 2$, $k = 1$, $p = 1$, $\xi = 1$, and $\phi = 2$.

an incentive for sellers to enter the market, by raising the expected income. Hence, when tightness is higher, more sellers offer goods for sale, and a larger fraction of these goods are sold, raising supply.

To wrap up our analysis of the market supply, we plot the supply curve in a tightness-quantity graph (figure 7.1B). The graph depicts the properties of the market supply that we have just derived.

7.2. Solution with endogenous capacity and fixed prices

We now solve the model with endogenous capacity in the simple case with fixed prices. We assume that the price for all goods is fixed, so the price norm simply imposes that the price of all goods is fixed to $p > 0$.

Then, to solve the model, we need to find the tightness θ that equalizes market demand and supply. With endogenous capacity, the condition is

$$y^d(\theta, p) = y^s(\theta, p), \tag{7.4}$$

where the market demand is the same as in the basic model, given by (5.17), and the market supply is specific to the model with endogenous capacity, given by (7.3).

The first step is to ensure that the solution indeed exists and is unique. We know from chapter 5 that $y^d(\theta, p)$ is decreasing from $y^d(0, p) > 0$ when $\theta = 0$ to 0 when $\theta = \bar{\theta}$. At the same time, we have seen in this chapter that $y^s(\theta, p)$ is increasing in tightness, starting from 0 when $\theta = 0$. Moreover, both y^d and y^s are continuous in tightness. Thus, we can be sure that equation (7.4) has a unique solution, since we can be sure that the curves

$y^d(\theta, p)$ and $y^s(\theta, p)$ cross once for $\theta \in (0, \bar{\theta})$. The crossing point is the solution of the model. This argument is the same really as in the model with exogenous capacity, which was illustrated in figure 5.3A. The only graphical difference is that the slope of the market supply is different.

Now that we have solved the model, we can look at comparative statics, starting with a negative demand shock: a decrease in preference for goods, a. From figure 5.4A, which continues to apply here for qualitative thinking, we see that a negative demand shock brings the demand curve inward, resulting in a lower value of both tightness θ and sales y. This was to be expected: as buyers value goods less, they visit fewer sellers, which reduces tightness and the amount of goods sold. The amount of slack in the market increases as it becomes harder for sellers to sell their goods.

Unlike in the case with fixed capacity, the drop in sales has two effects on sellers. First, there is lower participation in the market, since the participation rate falls when market tightness falls, as shown by (7.2). Sellers realize that the market is slacker, which means they are less likely to sell their goods, and that they expect a lower income. As a result, entering the market is not as profitable, so more sellers stay home. In addition, the share of goods that remain unsold, $1 - f(\theta)$, rises. So, sales fall because fewer goods are offered for sale and a smaller fraction of these goods are sold. The quantity of goods that are unsold, $[1 - f(\theta)]hk$, may go up or down depending on the relative response of the participation rate and the slack rate.

Next we turn to a negative supply shock. With endogenous capacity, several parameters can change and create such a supply shock, with slightly different implications. A negative supply shock could be a drop in the capacity k of each seller, or an increase in the outside option ξ of sellers. As (7.3) shows, both a lower k and a higher ξ lead to a lower market supply. From figure 5.4B, which continues to apply here too for qualitative analysis, we realize that a negative supply shock brings the supply curve inward, which results in a higher tightness θ but lower sales y. The logic is that fewer goods are supplied, so through the matching process, fewer goods are bound to be purchased. And because there is more competition among buyers for the fewer available goods, tightness is higher. Because tightness is higher, the slack rate falls in the market, as well as the number of goods that remain unsold (since fewer goods are placed for sale, and a smaller share of those are unsold).

What happens to the participation rate after the negative supply shock? The logic depends on the source of the shock. Let's consider first an increase in the utility from nonparticipation, ξ. Here the crux is that output and participation rate are related by $y = f(\theta)kh$, so $h = y/[f(\theta)k]$. We know that y is lower and $f(\theta)$ is higher, so clearly the participation rate is lower when ξ is higher. The reason is that, despite a higher tightness, participation has become less attractive after the increase in ξ. The increase in tightness

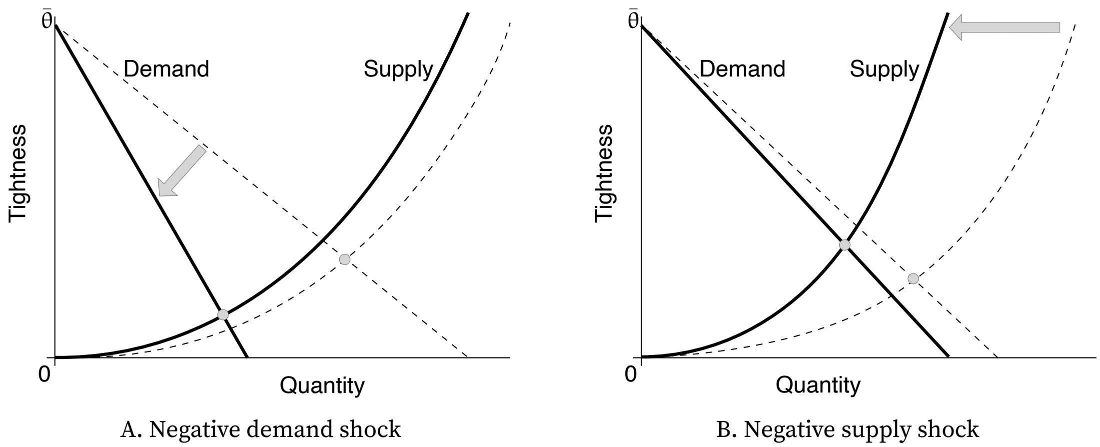


FIGURE 7.2. Comparative statics in the slackish market model with endogenous capacity and rigid prices

The market supply is given by (7.3). The market demand is given by (5.17). The price norm is given by (7.5). A negative demand shock is a reduction in the preference for goods a. A negative supply shock is a reduction in market capacity k or an increase in seller outside option ξ.

compensates a little bit the increase in the outside option, but not completely.

Next, let's look at a drop in individual capacity k. We see from (7.3) that the market supply is determined by $f(\theta)k$. Since output falls after the negative supply shock, and output is on the supply curve, it must be that $f(\theta)k$ falls—despite the increase in θ. From (7.2), we infer that the participation rate also falls when $f(\theta)k$ falls.

7.3. Solution with endogenous capacity and rigid prices

Just like in the case with fixed capacity, it is possible to generalize all the comparative-static results when prices are rigid instead of fixed. This is the most general form of a slackish market: with endogenous demand and capacity, as well as rigid prices that respond partially to demand and supply disturbances. Because of the generality, the analysis is a little bit more cumbersome. But the qualitative results are the same as with endogenous capacity and fixed prices, which are themselves the same as in the model with fixed capacity and fixed prices, so intuitions carry over naturally. The quantitative results (the elasticities of tightness with respect to various parameters) are more complex but are at the end of the day natural extensions of the elasticities with exogenous capacity and fixed prices.

With endogenous capacity, the rigid price must be slightly adjusted to capture the factors that influence market supply. Our rigid price norm will be

$$p^n = \left[a^{\frac{\phi}{\alpha+\phi}} \xi^{\frac{\alpha}{\alpha+\phi}} k^{-\frac{(1+\phi)\alpha}{\alpha+\phi}}\right]^{1-\gamma} \cdot \rho, \tag{7.5}$$

where $\gamma \in (0,1]$ determines the amount of rigidity of the price norm and $\rho > 0$ determines the level of the price norm. At $\gamma = 1$, the price is fixed and $p^n = \rho$. Notice that when $\phi \to \infty$, the price norm reduces to the price norm (6.1), which we used when capacity was exogenous.

It's natural to wonder where the rigid price norm comes from. Why does it take this specific form? It is built that way to capture all the forces that affect market tightness (which itself determines all other variables in the model). To build it, simply rewrite the supply-equals-demand condition and separate all the parameters on the left-hand side and all the terms that depend on tightness on the right-hand side. Using (5.17) and (7.3), we obtain the following equation:

$$(1-\alpha)^{1/\alpha} \cdot \frac{a^{1/\alpha} \xi^{1/\phi} k^{-\frac{1+\phi}{\phi}}}{p^{1/\alpha+1/\phi}} = f(\theta)^{\frac{1+\phi}{\phi}} \left[1+\tau(\theta)\right]^{1/\alpha-1}. \tag{7.6}$$

We then build the rigid price norm so the term $p^{1/\alpha+1/\phi}$ moves with $a^{1/\alpha}\xi^{1/\phi}k^{-\frac{1+\phi}{\phi}}$ but in a subdued fashion.

Because the price norm (7.5) is just a function of the parameters, and not of tightness, we can show that the model admits a unique solution, just like we did in the case with a fixed price. Indeed, the argument that we presented was valid for any price, so it is valid in particular if the price takes the value given by (7.5).

We can now consider a negative demand shock: a reduction in the preference for goods a. It leads to a reduction in market demand, as seen in (5.17), but also to a drop in price, as shown in (7.5). However, the response of the price norm to the demand shock is less than the shock, so even when we take into account the price reduction, the market demand falls. This can be seen because in the market demand, a appears only in the term a/p. Given the expression of the price norm (7.5), the term featuring a is proportional to

$$a^{1-\frac{(1-\gamma)\phi}{\alpha+\phi}}.$$

Since $1-\gamma \in (0,1)$ and $\phi/(\alpha+\phi) \in (0,1)$, the exponent on a is positive, even with the price response included. Thus, the market demand moves together with a, just like in the case with fixed prices. The drop in a also leads to a drop in market supply because of the reduction in price, which makes participation in the market less attractive.

We need to look at market supply and demand jointly to see what happens to tightness and output. To do that, we rewrite (7.6) by using the complete expression for the price norm, given by (7.5):

$$(1-\alpha)^{1/\alpha} \cdot \frac{\left[a^{1/\alpha} \xi^{1/\phi} k^{-\frac{1+\phi}{\phi}}\right]^{\gamma}}{\rho^{1/\alpha+1/\phi}} = f(\theta)^{\frac{1+\phi}{\phi}} \left[1+\tau(\theta)\right]^{1/\alpha-1}. \tag{7.7}$$

Overall, we see that when demand a drops, the left-hand side drops, since $\gamma > 0$. So the right-hand side must drop as well, to maintain the equality. We know that both f and $1+\tau$ are increasing functions of θ, and $1/\alpha > 1$, so the right-hand side is an increasing function of θ. This tells us that θ must fall when a falls. This comparative static is illustrated in figure 7.2A.

In fact, it is straightforward to compute the elasticity of tightness with respect to a from (7.7). It is even faster to go that way than to start from the supply-equal-demand condition, as we had done in chapter 6, because the parameters are all arranged together, and all the terms with tightness are grouped together. We see that the elasticity of the left-hand side with respect to a is simply

$$\epsilon_a^l = \frac{\gamma}{\alpha}.$$

Using (6.8), we see that the elasticity of the right-hand side with respect to tightness is

$$\epsilon_\theta^r = \frac{1+\phi}{\phi}(1-\eta) + \frac{1-\alpha}{\alpha}\eta\tau(\theta). \tag{7.8}$$

Now, implicit differentiation of (7.7) gives $\epsilon_a^l = \epsilon_\theta^r \epsilon_a^\theta$, so that

$$\epsilon_a^\theta = \frac{\gamma/\alpha}{\frac{1+\phi}{\phi}(1-\eta) + \frac{1-\alpha}{\alpha}\eta\tau(\theta)} = \frac{\gamma}{(1+1/\phi)(1-\eta)\alpha + (1-\alpha)\eta\tau(\theta)}.$$

We confirm that the elasticity is positive, so tightness drops when demand falls. At the same time, we see that the response is stronger when prices are more rigid (high γ). Indeed, if prices are more rigid, they cannot absorb the demand shock much, so tightness must absorb more of it. We also see that the elasticity is larger when the supply elasticity $1/\phi$ is lower.[1] This is because when the supply is less elastic, it responds less to the price reduction caused by the lower demand, so fewer sellers are discouraged from entering the market, which means that competition among sellers will be more drastic. In other words, tightness will fall more. And in fact we recognize that when $1/\phi \to 0$, the elasticity reduces to the elasticity with fixed capacity, given by (6.10).

We can repeat the same analysis with a negative supply shock: say a drop in seller capacity k. (We could similarly look at an increase in seller outside option ξ.) The reduced capacity leads to a reduction in market supply, naturally, despite the increase in price which attracts some new sellers into the market. This can be seen because in the market supply (7.3), k appears only in the term $k^{\frac{1+\phi}{\phi}} p^{\frac{1}{\phi}}$. Given the expression of the price norm

[1]The elasticity $1/\phi$ is akin to a Frisch elasticity of labor supply (Chetty et al. 2012).

(7.5), we realize that the term featuring k is proportional to

$$k^{\frac{1+\phi}{\phi}\left[1-(1-\gamma)\frac{\alpha}{\phi+\alpha}\right]}.$$

Since $1-\gamma \in (0,1)$ and $\alpha/(\alpha+\phi) \in (0,1)$, then the exponent on k is positive, even with the price response included, so the market supply moves together with k (just like in the case with fixed prices). The drop in k also leads to a drop in market demand because of the higher price.

We need to look at (7.7), which captures jointly market supply and demand, to see what happens to tightness and output. We see that the when capacity k drops, the left-hand side rises, since $-(1+\phi)\gamma/\phi < 0$. So the right-hand side must rise as well, to maintain the equality. The right-hand side is increasing in θ, so θ must rise when k falls. This comparative static is illustrated in figure 7.2B.

We can again without much effort compute the elasticity of tightness with respect to k from (7.7). The elasticity of the left-hand side with respect to k is

$$\epsilon^l_k = -\frac{(1+\phi)\gamma}{\phi}.$$

Now, implicit differentiation of (7.7) gives $\epsilon^l_k = \epsilon^r_\theta \epsilon^\theta_k$, where ϵ^r_θ is given by (7.8), so that

$$\epsilon^\theta_k = -\frac{\gamma}{(1-\eta)+\frac{1-\alpha}{\alpha}\cdot\frac{\phi}{1+\phi}\eta\tau(\theta)}.$$

We confirm that the elasticity is negative, so tightness is higher when capacity is reduced. Again, we see that the response of tightness is stronger (the amplitude $|\epsilon^\theta_k|$ of the elasticity is higher) when prices are more rigid (higher γ). The logic is the same as with the demand shock: if prices are more rigid, they cannot absorb the supply shock much, so tightness must absorb more of it. We also see that the amplitude of the elasticity is larger when the supply elasticity $1/\phi$ is higher. (A higher supply elasticity $1/\phi$ corresponds to a lower ϕ, so a lower term $\phi/(1+\phi)$ in the denominator, making the absolute value of the elasticity $|\epsilon^\theta_k|$ larger.) Intuitively, when the supply is less elastic, it responds less to the price increase caused by the lower supply, so fewer sellers are encouraged to enter the market—which would have alleviated the drop in seller capacity but here does not happen much. As a result, without much new entry, tightness will rise more. Again we realize here that when $1/\phi \to 0$, the elasticity reduces to the elasticity with fixed capacity, given by (6.12).

7.4. Summary

This chapter extends the slackish market model by making market capacity endogenous rather than exogenous. We allow potential sellers to decide whether to enter the market or not, based on their expected income from sales, which they compare against their utility from nonparticipation. The resulting market supply now depends on both the participation rate and the selling probability, making it increasing in both market tightness and price.

We analyze the model under both fixed and rigid prices and find that the comparative-static results remain consistent with those under exogenous capacity. Negative demand shocks reduce both tightness and sales while decreasing market participation, as sellers find entry less attractive due to lower expected selling probabilities. Negative supply shocks (whether from reduced individual capacity or increased nonparticipation utility) increase tightness but reduce sales and participation rates. Furthermore, when prices are more rigid, tightness absorbs more of the shocks. The same is true when supply elasticity is lower: tightness responds more because fewer sellers adjust their participation in response to price changes.

You would have noticed that here we did not look at the case with bargained prices. That is because bargained prices are exactly the same with endogenous capacity. Combined with the market demand that is unchanged, the bargained price determines tightness, as (6.6) shows. Hence demand shocks have no effect on the model, even with endogenous capacity, and supply shocks have no effect on tightness but affect output, and here the participation rate, in a straightforward way.

Bibliography

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. 2012. “Does Indivisible Labor Explain the Difference between Micro and Macro Elasticities? A Meta-Analysis of Extensive Margin Elasticities.” *NBER Macroeconomics Annual* 27 (1): 1–56. `https://doi.org/10.1086/669170`.